INTRODUCTION TO QUANTUM EFFECTS IN GRAVITY

This is the first introductory textbook on quantum field theory in gravitational backgrounds intended for undergraduate and beginning graduate students in the fields of theoretical astrophysics, cosmology, particle physics, and string theory. The book covers the basic (but essential) material of quantization of fields in expanding universe and quantum fluctuations in inflationary spacetime. It also contains a detailed explanation of the Casimir, Unruh, and Hawking effects, and introduces the method of effective action used for calculating the backreaction of quantum systems on a classical external gravitational field.

The broad scope of the material covered will provide the reader with a thorough perspective of the subject. Complicated calculations are avoided in favor of simpler ones, which still contain the relevant physical concepts. Every major result is derived from first principles and thoroughly explained. The book is self-contained and assumes only a basic knowledge of general relativity. Exercises with detailed solutions are provided throughout the book.

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Preface

This book is an expanded and reorganized version of the lecture notes for a course taught at the Ludwig-Maximilians University, Munich, in the spring semester of 2003. The course is an elementary introduction to the basic concepts of quantum field theory in classical backgrounds. A certain level of familiarity with general relativity and quantum mechanics is required, although many of the necessary concepts are introduced in the text.

The audience consisted of advanced undergraduates and beginning graduate students. There were 11 three-hour lectures. Each lecture was accompanied by exercises that were an integral part of the exposition and encapsulated longer but straightforward calculations or illustrative numerical results. Detailed solutions were given for all the exercises. Exercises marked by an asterisk (*) are more difficult or cumbersome.

The book covers limited but essential material: quantization of free scalar fields; driven and time-dependent harmonic oscillators; mode expansions and Bogolyubov transformations; particle creation by classical backgrounds; quantum scalar fields in de Sitter spacetime and the growth of fluctuations; the Unruh effect; Hawking radiation; the Casimir effect; quantization by path integrals; the energy-momentum tensor for fields; effective action and backreaction; regularization of functional determinants using zeta functions and heat kernels. Topics such as quantization of higher-spin fields or interacting fields in curved spacetime, direct renormalization of the energy-momentum tensor, and the theory of cosmological perturbations are left out.

The emphasis of this course is primarily on concepts rather than on computational results. Most of the required calculations have been simplified to the barest possible minimum that still contains all relevant physics. For instance, only free scalar fields are considered for quantization; background spacetimes are always chosen to be conformally flat; the Casimir effect, the Unruh effect, and the Hawking radiation are computed for massless scalar fields in suitable

$1+1$-dimensional spacetimes. Thus a fairly modest computational effort suffices to explain important conceptual issues such as the nature of vacuum and particles in curved spacetimes, thermal effects of gravitation, and backreaction. This should prepare students for more advanced and technically demanding treatments suggested below.

The authors are grateful to Josef Gaßner and Matthew Parry for discussions and valuable comments on the manuscript. Special thanks are due to Alex Vikman who worked through the text and prompted important revisions, and to Andrei Barvinsky for his assistance in improving the presentation in the last chapter.

The entire book was typeset with the excellent LyX and TeX document preparation system on computers running Debian GNU/Linux. We wish to express our gratitude to the creators and maintainers of this outstanding free software.

Suggested literature

The following books offer a more extensive coverage of the subject and can be studied as a continuation of this introductory course.

N. D. BIRRELL and P. C. W. DAVIES, *Quantum Fields in Curved Space* (Cambridge University Press, 1982).

S. A. FULLING, *Aspects of Quantum Field Theory in Curved Space-Time* (Cambridge University Press, 1989).

A. A. GRIB, S. G. MAMAEV, and V. M. MOSTEPANENKO, *Vacuum Quantum Effects in Strong Fields* (Friedmann Laboratory Publishing, St. Petersburg, 1994).