Physics H190
Spring 2013
Homework 5
Due Friday, March 8, 2013 at 5pm

Change in Due Date: One student asked that I change the due date of the homework to Friday, which is reasonable since it will give you a whole week to do the homework. I will place an envelope outside my office, 449 Birge, for you to put the homework in. The homework is due at 5pm. Also, I want the office hours to be the day before the homework is due, since that's when you're most likely to need them. So I'm changing the office hours. The main office hour will be 4pm on Thursday next week, and I'll set up a secondary one in class next Wednesday.

Reading Assignment: Please read the posted notes on tensor analysis (Appendix E from Physics 221), Secs. 1–13. Also please read over Chapters 3 and 4 of Mukhanov and Winitzski, not trying to follow it in detail, but to get some of the ideas.

1. Consider the transformation properties of tensors under general coordinate transformations. Show that it is meaningful for a completely covariant or completely contravariant second rank tensor to be symmetric or antisymmetric, for example, $C^{ij} = C^{ji}$ or $D_{ij} = -D_{ji}$, that is, show that if such a property holds in one coordinate system it holds in all. Then show that in general there is no meaning to the symmetry or antisymmetry of a mixed tensor, that is, such a property is not invariant under coordinate transformations.

2. As shown in the notes, the covariant metric tensor, defined by

$$ds^2 = g_{ij}\, dx^i\, dx^j, \tag{1}$$

does indeed transform as a covariant tensor. Now we define g^{ij} as the inverse matrix of g_{ij} in one coordinate system x^i. Next we compute g'^{ij} in a new coordinate system x'^i in two different ways. The first way is to transform g_{ij} to g'_{ij} using the covariant tensor tranformation law, then we invert the matrix g'_{ij}. The other way is we take g^{ij} in the original coordinate system and tranform it to the new coordinates using the transformation law for contravariant tensors. Show that these two procedures give the same answer (hence, g^{ij} transforms as a contravariant tensor). What do we get if we take the covariant metric tensor g_{ij} and raise both indices? What is the mixed version of the metric tensor $g^i{}_j$ or $g_i{}^j$, in which only one index has been raised?

The questions submitted for last Wednesday were very good, and I will answer as many of them as I can, adding them to the end of this homework assignment in the next several days. I will also post more chapters of Mukhanov and Winitzski.