Physics H190 Spring 2005 Homework 6 Due Wednesday, March 9, 2005

Reading Assignment: Read Richtmeyer, Kennard and Lauritsen, 106–113. The lecture notes for this week cover most of the material given in lecture, but not all of it.

1. The hyperbola billiard (see figure) is a free particle in the x-y plane, confined by hard walls on the x- and y-axes and on the hyperbola $y = a^2/x$, where a is some length. It is a particle in a 2-dimensional box of a particular shape. Inside the box, the Hamiltonian is

$$H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m}.$$
 (1)

It is obvious from the figure that the particle shown feels an effective repulsive force in the x-direction, which causes it to reach an x-turning point and turn around. Notice that when the particle is far out on the x-leg of the hyperbola, the height of the wall $(y = a^2/x)$ is nearly constant from one bounce to the next. You can think of the y-motion as a "yoscillator," that is, a particle in a one-dimensional box, whose height is a slow function of x. Find a conserved quantity and use it to express the Hamiltonian in terms of x, p_x , and the conserved quantity. The result is an effective Hamiltonian for the x-motion. What is the repulsive force, as a function of x?