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that in the long and honourable history of the Royal Society no mistake more disastrous in its actual consequences for the progress of science and the reputation of British science than the rejection of Waterston's papers was ever made. . . . There is every reason for believing that had the papers been published physical chemistry and thermodynamics would have developed mainly in this country and along much simpler, more correct, and more intelligible lines than those of their actual development."[54] More than forty years had to pass until the 1845 memoir, under its original title, was finally published in the *Philosophical Transactions*.[55] With reference to Waterston's enunciation of the equipartition theorem, Lord Rayleigh, through whose efforts the paper was published, declared in the introduction: "The omission to publish it at the time was a misfortune which probably retarded the development of the subject by ten or fifteen years. It is singular that Waterston appears to have advanced no claim for subsequent publication, whether in the *Transactions* of the Society, or through some other channel. At any time since 1860 reference would naturally have been made to Maxwell, and it cannot be doubted that he would have at once recommended that everything possible should be done to atone for the original failure of appreciation."

Rayleigh's reference to Maxwell alludes to his paper "Illustrations of the dynamical theory of gases,"[56] in which Maxwell elaborated some conclusions submitted one year earlier at the Aberdeen meeting of the British Association.[57] There he gave his first formulation of the equipartition theorem as follows: "Two different sets of particles will distribute their velocities, so that their *vires vivae* will be equal." At first he considered only the case of "smooth spherical particles" but later, in a corollary, extended the theorem to the case of a mixture of particles of any form and included rotation.

In 1868 the theorem was further generalized by Boltzmann, who proved[58] its validity also for particles which are not necessarily rigid but have a number of internal degrees of freedom. Finally, Maxwell[59] removed certain restrictions on the interaction among particles and showed, using generalized Lagrangian coordinates for systems with an arbitrary number of degrees of freedom, that the equipartition of energy holds even if "the

[54] REF. 50, p. lxv.

[55] *Philosophical Transactions of the Royal Society of London, 183*, 1–79 (1892); reprinted in *The Collected Scientific Papers* (REF. 50), pp. 207-319.

[56] *Philosophical Magazine 20*, 21–37 (1860); reprinted in *The Scientific Papers of James Clerk Maxwell*, edited by W. D. Niven (Cambridge University Press, 1890; republished by Dover, New York), vol. 1, pp. 378–409.

[57] J. C. Maxwell, "On the dynamical theory of gases," *British Association Reports, Aberdeen, 29*, 9 (1859).

[58] L. Boltzmann, "Studien über das Gleichgewicht der lebendigen Kraft zwischen bewegten materiellen Punkten," *Wiener Berichte 58*, 517–560 (1868).

[59] J. C. Maxwell, "On Boltzmann's theorem on the average distribution of energy in a system of material points," *Transactions of the Cambridge Philosophical Society 12*, 547–570 (1878); *Scientific Papers* (REF. 56), vol. 2, pp. 713–741.

material points may act on each other at all distances, and according to any law which is consistent with the conservation of energy . . . the only assumption which is necessary for the direct proof is that the system, if left to itself in its actual state of motion, will, sooner or later, pass through every phase which is consistent with the equation of energy."

Yet the very generalizations of the theorem jeopardized its validity. For, as Tait[60] put it in his critical examination of Boltzmann's approach and in his search for an unassailable proof of the theorem: "There can be no doubt that each individual particle of a gas has a very great number of degrees of freedom besides the six which it would have if it were rigid; the examination of its spectrum while incandescent proves this at once. But if all these degrees of freedom are to share the whole energy (on the average) equally among them, the results of theory will no longer be consistent with our experimental knowledge of the two specific heats of a gas, and the relations between them." A still more drastic description of the problems raised by the theorem was given by Lord Kelvin[61] in a lecture at the Royal Institution on April 27, 1900, when he said: "The beauty and clearness of the dynamical theory, which asserts heat and light to be modes of motion, is at present obscured by two clouds. The first . . . involved the question, How could the earth move through an elastic solid, such as essentially is the luminiferous ether? The second is the Maxwell-Boltzmann doctrine of partition of energy."

It should be obvious from these remarks that the scientific literature at the end of the nineteenth century, both in England and on the continent, contained numerous articles dealing with the doctrine of energy equipartition, and there cannot be any doubt that Planck must have had knowledge of the equipartition theorem.

However, fortunately for the future development of physics, Planck did not make use of the theorem. It is hard to say whether it was in view of these difficulties or because of his unfamiliarity with the Boltzmann-Gibbs methods of statistical mechanics[62] or his profound aversion to the molecular approach[63] or, finally, because of his strong conviction in the power of thermodynamic reasoning based on the concept of entropy. One thing is certain: had he used the equipartition theorem at this stage of

[60] P. G. Tait, "On the foundations of the kinetic theory of gases," *Transactions of the Royal Society of Edinburgh, 33*, 65–95, 251–277 (1886; all four parts of this paper, published between 1886 and 1891, are reprinted in *Scientific Papers by Peter Guthrie Tait* (Cambridge University Press, 1900), vol. 2, pp. 124–208.

[61] Lord Kelvin, "Nineteenth century clouds over the dynamical theory of heat and light," *Philosophical Magazine 2*, 1–40 (1901); *Baltimore Lectures on Molecular Dynamics and the Wave Theory of Light* (London, Baltimore, 1904), pp. 486–527.

[62] This reason is given by Werner Heisenberg, *Das Plancksche Wirkungsquantum* (Walter de Gruyter, Berlin, 1945); reprinted in *Max Planck, Erinnerungen* (W. Keiper, Berlin, 1948), pp. 69–82.

[63] Arguments for the plausibility of this reason are adduced by Martin J. Klein, "Max Planck and the beginning of the quantum theory," *Archives for History of Exact Sciences 1*, 459–479 (1962).

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[64] REF. 42, 5th communication.
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his work, he would necessarily have arrived at the Rayleigh-Jeans law of radiation, which is incompatible with experience, and would probably have given up further research on this problem. Instead, adopting what he later called the "thermodynamic approach," Planck[64] defined the entropy S of an oscillator by the equation

$$S = \frac{U}{a\nu} \log \left(\frac{U}{eb\nu} \right) \tag{1.7}$$

where a and b are constants and e is the base of natural logarithms. The quantity $\partial^2 S/\partial U^2$, which proved important in connection with the principle of the increase of entropy, thus satisfied the equation

$$\frac{\partial^2 S}{\partial U^2} = \frac{\text{const}}{U} \tag{1.8}$$

The reasoning which led Planck to this apparently arbitrary definition of S in terms of U and ν may be reconstructed on the basis of certain remarks found at the end of his fifth communication.[65] From (1.6) and guided by the form of (1.5), Planck obtained $U = C\nu \exp(-\beta\nu/T)$, where C is a constant. Solving this equation for T^{-1}, which according to thermodynamics equals, at constant volume, $\partial S/\partial U$, Planck obtained (1.7) by simple integration. Consistent with his assumption concerning the irreversibility associated with "natural radiation," he then showed that the "total electric entropy" $S_t = \sum S + \int s\, d\tau$, where the summation extends over all oscillators and the integration over all volume elements $d\tau$ of the radiation field with entropy density s, is a function of state which increases in time and reaches a maximum at equilibrium. Planck now assumed that a small amount of energy passes from one oscillator of frequency ν, entropy S, and energy U, to another of frequency ν', entropy S', and energy U'. The entropy and energy principles require that $\delta S_t = \delta S + \delta S' = 0$ and $\delta U + \delta U' = 0$, which in view of (1.7) lead to the equation $-(a\nu)^{-1} \log (U/b\nu) = -(a\nu')^{-1} \log (U'/b\nu')$. Hence the expression on the left-hand side of this equation is a constant for all oscillators considered and therefore, in virtue of (1.6), a common parameter of u_ν for all ν. Setting this expression—which has just been shown to be a function only of T—equal to T^{-1}, Planck obtained $U = b\nu \exp(-a\nu/T)$, which, in combination with (1.6), yielded Wien's radiation law (1.5). Being fully aware that the result was determined by the particular choice of (1.7), Planck contended that an equation only of the form of (1.5) does lead to an expression for S which satisfies the entropy principle.

[64] REF. 42, 5th communication.

[65] In section 23 of this paper Planck refers to a "retrogressive computation" of the entropy from the energy distribution law: ". . . berechnet man daraus rückwärts den Ausdruck der Entropie . . . ." *Physikalische Abhandlungen und Vorträge* (REF. 42), vol. 1, p. 596.

Before the turn of the century, however, the unrestricted validity of Wien's radiation law (1.5) was seriously challenged. Lummer and Pringsheim[66] recorded systematic deviations for smaller frequencies; when additional measurements[67] in the range from 12 to 18 μ confirmed their suspicion, they had the courage to declare: "It has been demonstrated that black-body radiation is not represented, in the range of wavelengths measured by us, by the Wien-Planck spectral equation."[68] In addition to these objections based on experiment also Planck's theoretical procedure in deriving equation (1.5) became the target of severe criticisms.[69] No wonder that Thiesen,[70] Lummer and Jahnke,[71] and Lummer and Pringsheim[72] proposed new distribution laws to fit also the experimental data obtained for longer waves.

Meanwhile Lord Rayleigh, in a two-page paper "Remarks upon the law of complete radiation,"[73] published in June, 1900, showed that the equipartition theorem of statistical mechanics, if applied to the electromagnetic vibrations of cavity radiation, led necessarily to a formula radically different from (1.5). Rayleigh, an expert in the mathematical treatment of standing waves, as he had already shown in his *Theory of Sound*,[74] computed the number N_λ of modes of free electromagnetic vibrations per unit volume in an enclosure and per unit range of wavelength at λ and found[75]—if we consider Jean's subsequent correction[76] of Rayleigh's result—that N_λ is equal to $8\pi/\lambda^4$. Assuming that the average energy of each mode at temperature T, according to the equipartition theorem, is

[66] REF. 24 (1899).

[67] O. Lummer and E. Pringsheim, "Über die Strahlung des schwarzen Körpers für lange Wellen," *Verhandlungen der Deutschen Physikalischen Gesellschaft 2*, 163–180 (1900).

[68] *Ibid.*, p. 171.

[69] W. Wien, "Les lois théoriques du rayonnement," in *Rapports Présentés au Congrès International de Physique* (Gauthier-Villars, Paris, 1900), vol. 2, pp. 23–40. Cf. also REF. 67, p. 166.

[70] REF. 11.

[71] O. Lummer and E. Jahnke, "Über die Spectralgleichung des schwarzen Körpers und des blanken Platins," *Annalen der Physik 3*, 283–297 (1900).

[72] O. Lummer and E. Pringsheim, "Über die Strahlung des schwarzen Körpers für lange Wellen," *Verhandlungen der Deutschen Physikalischen Gesellschaft 2*, 163–180 (1900).

[73] *Philosophical Magazine 49*, 539–540 (1900); *Scientific Papers of Lord Rayleigh* (Cambridge University Press, 1902–1910; Dover, New York, 1964), vol. 4, pp. 483–485.

[74] John William Strutt, Baron Rayleigh, *The Theory of Sound* (Macmillan, London, 1877–1878; 2d ed. 1929; Dover, New York, 1945).

[75] In the second volume of *The Theory of Sound* (*ibid.*), sec. 267, Rayleigh had solved the same problem for acoustical vibrations. See chap. 13, which begins with the words: "We will now inquire what vibrations are possible within a closed rectangular box . . ." (1878 ed., p. 65; 1929 ed., p. 67; 1945 ed., p. 69).

[76] J. H. Jeans, "On the partition of energy between matter and ether," *Philosophical Magazine 10*, 91–98 (1905). Jeans pointed out that the original Rayleigh formula had to be divided by 8 since only the octant of positive integers, and not the whole sphere, has to be taken into account. Thus, Jeans's contribution to the "Rayleigh-Jeans" law was only the statement: "It seems to me that Lord Rayleigh has introduced an unnecessary factor 8 by counting negative as well as positive values of his integers." *Ibid.*, p. 98, Postscript, added June 7.

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$(R/N)\,T$, where R is the universal gas constant and N Avogadro's number, or kT, where k is Boltzmann's constant, introduced at that time by Planck, Lord Rayleigh obtained for the energy density per unit interval of wavelength $u_\lambda = 8\pi kT/\lambda^4$ or equivalently

$$u_\nu = \frac{8\pi\nu^2 kT}{c^3} \tag{1.9}$$

This formula, the "Rayleigh-Jeans radiation law," agreed, of course, with Wien's displacement law (1.3). It also agreed with all experimental data in the region of extremely low frequencies, just where Wien's radiation law failed. On the other hand, it was immediately clear that (1.9) must be wrong for high frequencies. It assigned no maximum to u_ν or B_ν, contrary to experience, and led—in view of the unlimited increase for higher frequencies—to a divergent integral for the total energy density u, a situation which was later, following Ehrenfest,[77] referred to as the "ultraviolet catastrophe."

However, that for low frequencies and high temperatures u was proportional to T, as required by (1.9) in contrast to (1.5), had meanwhile been established irrefutably in a series of measurements carried out by Rubens and Kurlbaum.[78] The importance of these measurements for the future development of quantum theory is best characterized by Planck himself, who admitted that "without the intervention of Rubens the formulation of the radiation law and consequently the foundation of quantum theory would have perhaps taken place in a totally different manner and perhaps even not at all in Germany."[79] A few days before presenting their results to the Berlin Academy,[80] which was to convene on October 25, 1900, Rubens and Kurlbaum reported their observations to Planck. Convinced by their report of the inadequacy of Wien's radiation law, Planck realized that his reasoning leading to the Wien formula had to be revised so that it would lead to a new formula which for large ν and small T agrees with Wien's expression but for small ν and large T reduces to a proportionality of u_ν with T. Planck's point of departure was, of course, definition (1.7).

[77] P. Ehrenfest, "Welche Züge der Lichtquantenhypothese spielen in der Theorie der Wärmestrahlung eine wesentliche Rolle," *Annalen der Physik 36*, 91–118 (1911). The fourth chapter of this paper is entitled "Die Vermeidung der Rayleigh-Jeans-Katastrophe im Ultravioletten," where the term "ultraviolet catastrophe" appeared for the first time. Reprinted in P. Ehrenfest, *Collected Scientific Papers*, edited by Martin J. Klein (North-Holland Publishing Co., Amsterdam; Interscience, New York, 1959), pp. 185–212.

[78] H. Rubens and F. Kurlbaum, "Anwendung der Methode der Reststrahlen zur Prüfung des Strahlungsgesetzes," *Annalen der Physik 4*, 649–666 (1901).

[79] "Gedächtnisrede des Hrn Planck auf Heinrich Rubens," *Berliner Berichte 1923* (June 28), p. cxi.

[80] "Über die Emission langwelliger Wärmestrahlen durch den schwarzen Körper bei verschiedenen Temperaturen," *Berliner Berichte 1900*, pp. 929–941.

Since under the latter conditions with u_ν, according to (1.6), also U has to be proportional to T and since $\partial S/\partial U = T^{-1}$, Planck inferred that S is proportional to log U or

$$\frac{\partial^2 S}{\partial U^2} = \frac{\text{const}}{U^2} \tag{1.10}$$

whereas for the former conditions (1.8) has to remain valid. Compromising, therefore, between (1.8) and (1.10), Planck assumed[81]

$$\frac{\partial^2 S}{\partial U^2} = \frac{a}{U(U+b)} \tag{1.11}$$

which, in fact, reduces for small values of U to (1.8) and hence to Wien's law, and for large values of U to (1.10) and hence to the Rubens-Kurlbaum results.

This interpolation, though mathematically a mere trifle, was one of the most significant and momentous contributions ever made in the history of physics. Not only did it lead Planck, in his search for its logical corroboration, to the proposal of his elementary quantum of action and thus initiate the early development of quantum theory, as we shall see presently; it also contained certain implications which, once recognized by Einstein, affected decisively the very foundations of physics as well as their epistemological presuppositions. Never in the history of physics was there such an inconspicuous mathematical interpolation with such far-reaching physical and philosophical consequences.

Now, from this interpolation, Eq. (1.11), Planck deduced that

$$\frac{1}{T} = \frac{\partial S}{\partial U} = a' \log \frac{U+b}{U}$$

or

$$U = \frac{b}{\exp(1/a'T) - 1}$$

where $a' = -a/b$ and b are, of course, still functions of ν. To find their dependence on ν, Planck referred to (1.5) and (1.6) and obtained

$$U = \nu\Phi\left(\frac{\nu}{T}\right) \tag{1.12}$$

where $\Phi(\nu/T)$ is a function of ν/T. Hence, he concluded,

$$U = \frac{\text{const}\ \nu}{\exp(c'\nu/T) - 1}$$

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[83] *Ibid.*, vol. 3, p. 263.

[84] *Ibid.*, p. 125.

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$$E_\lambda = \frac{C\lambda^{-5}}{\exp\,(c/\lambda T) - 1}$$

or

$$u_\nu = \frac{A\nu^3}{\exp\,(B\nu/T) - 1} \qquad (1.13)$$

where c', C, A, and B are constants.

Planck obtained this result just in time to prepare an extended "comment" ("Diskussionsbemerkung") to follow Kurlbaum's report to the German Physical Society, which met on October 19, 1900.

In this "comment," published under the title "On an improvement of Wien's radiation law,"[82] Planck announced formula (1.13), the formulation of what was later called "Planck's law of radiation." For the time being, it was an empirical formula since its basic assumption (1.11) had no rigorous theoretical justification. But it seemed to be a correct formula. Rubens,[83] who through the night following the Academy session checked it against his experimental results, reported complete agreement, as did Lummer and Pringsheim after correcting their own errors of calculation a short time afterward.

To change the status of (1.11) from that of a "lucky guess" ("eine glücklich erratene Interpolationsformel")[84] to that of a "statement of real physical significance," Planck ultimately found it necessary to abandon his "thermodynamic approach" and to turn to Boltzmann's probabilistic conception of entropy.[85] Writing S_N for the entropy of a system of N oscillators of frequency ν, Planck, apparently following Boltzmann, posited $S_N = k \log W$, where W is the number of distributions compatible with the energy of the system. In order to determine W, Planck had to assume that the total energy $U_N = NU$ consists of an integral number P of "energy elements" ϵ ("Energie-elemente") so that $U_N = P\epsilon$, for the traditional conception of U_N as a continuous magnitude would not have admitted a combinatorial procedure for the determination of W. Since it is exactly at this point that the methodological requirement for a combinatorial procedure motivated Planck's introduction of the quantum of

[82] "Über eine Verbesserung der Wienschen Spektralgleichung," *Verhandlungen der Deutschen Physikalischen Gesellschaft 2*, 202–204 (1900); the paper was read at the meeting of the German Physical Society on Oct. 19, 1900: *Physikalische Abhandlungen und Vorträge* (REF. 42), vol. 1, pp. 687–689.

[83] *Ibid.*, vol. 3, p. 263.

[84] *Ibid.*, p. 125.

[85] L. Boltzmann, "Über die Beziehung zwischen dem zweiten Hauptsatz der mechanischen Wärmetheorie und der Wahrscheinlichkeitsrechnung respective den Sätzen über das Wärmegleichgewicht," *Wiener Berichte 76*, 373–435 (1877). Reprinted in L. Boltzmann, *Wissenschaftliche Abhandlungen* (Barth, Leipzig, 1909), vol. 2, p. 164.

action which, in its turn, led eventually to the development of quantum theory and its departure from the principles of classical physics, it is appropriate to quote Planck's first explicit reference to h: "Now we have to consider the distribution of the energy U_N among the N resonators of frequency ν. If U_N were regarded as an infinitely divisible quantity, the distribution could be performed in an infinite number of ways. We consider, however—and this is the cardinal point of the whole computation—U_N as composed of a finite number of discrete equal parts and employ for this purpose the natural constant $h = 6.55 \times 10^{-27}$ erg sec. This constant multiplied by the common frequency ν of the resonators gives the energy element ϵ in ergs, and by dividing U_N by ϵ we obtain the number P of energy elements which are distributed among the N resonators."[86]

Interpreting W in the equation $S_N = k \log W$ as the number of possible ways of distributing P energy elements ϵ among N oscillators, Planck[87] obtained

$$W = \frac{(N + P - 1)!}{(N - 1)!P!}$$

[86] "... Nun ist noch die Verteilung der Energie U_N auf die N Resonatoren mit der Schwingungszahl ν vorzunehmen. Wenn U_N als unbeschränkt teilbare Größe angesehen wird, ist die Verteilung auf unendlich viele Arten möglich. Wir betrachten aber—und dies ist der wesentliche Punkt der ganzen Berechnung—U_N als zusammengesetzt aus einer ganz bestimmten Anzahl endlicher gleicher Teile und bedienen uns dazu der Naturconstante $h = 6.55 \times 10^{-27}$ erg sec. Diese Constante mit der gemeinsamen Schwingungszahl ν der Resonatoren multipliziert ergiebt das Energieelement ϵ in erg, und durch Division von U_N durch ϵ erhalten wir die Anzahl P der Energieelemente, welche unter die N Resonatoren zu verteilen sind." *Berliner Berichte* (Dec. 14, 1900); *Physikalische Abhandlungen und Vorträge* (REF. 42), vol. 1, pp. 700–701. Planck used the letter E instead of our U_N.

[87] A very simple proof of the combinatorial formula was given by P. Ehrenfest and H. Kamerlingh Onnes in their paper "Vereenvoudigde afleiding van de formule uit de combinatieleer, welke Planck aan zijne theorie der straling ten groundslag heeft gelegd," *Verslag van de Gewone Vergaderingen der Wis- en Natuurkundige Afdeeling, Koninklijke Akademie van Wetenschappen te Amsterdam 23*, 789–790 (1914); the paper had an appendix: "De tegenstelling tusschen de hypothese der energietrappen van Planck en de hypothese der energiequanta van Einstein," *ibid.*, 791–792. The English version, "Simplified deduction of the formula from the theory of combinations which Planck uses as the basis of his radiation theory," appeared in the *Proceedings of the Amsterdam Academy 17*, 870–872 (1914), and the appendix "The contrast between Planck's hypothesis of the energy-grades and Einstein's hypothesis of the energy quanta," in *ibid.*, 872–873; both are reprinted in P. Ehrenfest, *Collected Scientific Papers* (REF. 77), pp. 353–356. Cf. also "Vereinfachte Ableitung der kombinatorischen Formel, welche der Planckschen Strahlungstheorie zugrunde liegt," *Annalen der Physik 46*, 1021–1022 (1915), with the appendix "Der Gegensatz zwischen der Energiestufenhypothese von Planck und der Energiequantenhypothese von Einstein," *ibid.*, 1022–1024. The possible distributions are represented by a set of P identical symbols for the P energy elements and by a different set of $N - 1$ identical symbols for "partitions." The $(N + P - 1)!$ possible permutations of all symbols, divided by the $P!$ permutations of the energy elements and the $(N - 1)!$ permutations of the partitions, represent all possible modes of distribution. It is tacitly assumed, as we see, that the energy elements are indistinguishable or, in other words, that the exchange of any two energy elements, even if they belong to different resonators, does not produce a new mode of distribution.

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[88] "Die Entstehung und bis
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and by the use of Stirling's formula

$$W = \frac{(N+P)^{N+P}}{N^N P^P}$$

so that

$$S_N = k[(N+P)\log(N+P) - N\log N - P\log P]$$

or finally

$$S_N = kN\left[\left(1+\frac{U}{\varepsilon}\right)\log\left(1+\frac{U}{\varepsilon}\right) - \frac{U}{\varepsilon}\log\frac{U}{\varepsilon}\right] \tag{1.14}$$

Since the entropy $S = S_N/N$ of a single oscillator did indeed satisfy Eq. (1.11), Planck felt sure he was on the right track. From $\partial S/\partial U = 1/T$ he now obtained for the average energy U of the oscillators of frequency ν

$$U = \frac{\epsilon}{\exp(\epsilon/kT) - 1} \tag{1.15}$$

which is compatible with his previous result $U = \nu\Phi(\nu/T)$ only if $\epsilon = h\nu$, where h is a constant independent of ν. Finally, in view of Eq. (1.6) Planck arrived at his famous radiation law

$$u_\nu = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{\exp(h\nu/kT) - 1} \tag{1.16}$$

in agreement with Eq. (1.13). Integrating Eq. (1.16) over all frequencies, Planck obtained the Stefan-Boltzmann law and established a relation between k^4/h^3 and σ; calculating the frequency at which u_ν reaches a maximum, he confirmed Eq. (1.4) and related h/k to b. From the known values of σ and b Planck computed the numerical value of the constant of action and found $h = 6.55 \times 10^{-27}$ erg sec. In addition he computed $k(1.346 \times 10^{-16}$ erg deg^{-1}) and, with the help of the gas constant R, Avogadro's number (6.175×10^{23} mole^{-1}). Finally, from Faraday's constant he determined the elementary unit charge $e(4.69 \times 10^{-10}$ esu).

These results were obtained within a period of about eight weeks which Planck described two decades later: "After a few weeks of the most strenuous work of my life, the darkness lifted and an unexpected vista began to appear."[88] At the meeting of the German Physical Society on December 14, 1900, a date which is often regarded as the "birthday of

[88] "Die Entstehung und bisherige Entwicklung der Quantentheorie," *Nobel Prize Lecture*, delivered to the Royal Swedish Academy, Stockholm, on June 2, 1920. *Physikalische Abhandlungen und Vorträge* (REF. 42), vol. 3 pp. 121–134; English translation in Planck, *A Survey of Physics* (Methuen, London, 1922; Dover, New York, 1960).

quantum theory,"[89] Planck read his historic paper "On the theory of the energy distribution law of the normal spectrum,"[90] in which he presented these results and introduced the "universal constant h," destined to change the course of theoretical physics.

It should be noted that Planck's combinatorial approach differed from Boltzmann's probabilistic method[91] in so far as Planck associated W with S_N at the equilibrium state without maximizing it. For Planck W was merely the total number of possible complexions and not, as for Boltzmann, the number of possible complexions corresponding to the macro state which can be realized by the largest number of complexions. The reason for this deviation was probably the fact, as already pointed out by Rosenfeld,[92] that Planck's actual point of departure was the expression (1.14) for S, to fit conjecture (1.11), and that therefore W, to satisfy the equation $S_N = k \log W$, necessarily had to be of the form $(N + P)^{N+P}/N^N P^P$, which, because of its similarity to the well-known combinatorial formula[93] $(N + P - 1)!/P!(N - 1)!$, prompted him to adopt the combinatorial procedure the way he did.

It is also interesting to note that nowhere in this paper, nor in any other of his early writings, did Planck bring into prominence the fundamental fact that U is an integral multiple of $h\nu$. At that time Planck apparently was not yet quite sure whether his introduction of h was merely a mathematical device or whether it expressed a fundamental innovation of profound physical significance. In an unpublished letter[94] (1931), addressed to R. W. Wood, Planck described in detail the psychological motives which led him to the postulate of energy quanta: he called it "an act of desperation," done because "a theoretical explanation *had* to be supplied at all cost, whatever the price." As he admitted later in his *Autobiography*,[95] he was dissatisfied with his own approach and attempted repeatedly, though unsuccessfully, to fit the introduction of h somehow ("irgendwie") into the framework of classical physics. On the other hand, his son reported how his father, on long walks through the Grunewald, a forest in the suburbs of

[89] E.g., by Max von Laue in his Memorial Address, delivered at Planck's funeral in the Albani Church, Göttingen, on Oct. 7, 1947. Cf. *Physikalische Abhandlungen und Vorträge* (REF. 42), vol. 3, p. 419; *Scientific Autobiography* (Philosophical Library, New York, 1949), p. 10.

[90] "Zur Theorie des Gesetzes der Energieverteilung im Normalspektrum," *Verhandlungen der Deutschen Physikalischen Gesellschaft 2*, 237–245 (1900); "Über das Gesetz der Energieverteilung im Normalspektrum," *Annalen der Physik 4*, 553–563 (1901); *Physikalische Abhandlungen und Vorträge* (REF. 42), vol. 1, pp. 717–727.

[91] Boltzmann's method would also have led to (1.14).

[92] L. Rosenfeld, "La première phase de l'évolution de la Théorie des Quanta," *Osiris 2*, 149–196 (1936).

[93] This formula had already appeared in Boltzmann's paper referred to in REF. 85.

[94] "Kurz zusammengefasst kann ich die ganze Tat als einen Akt der Verzweiflung bezeichnen." The letter (Oct. 7, 1931) is deposited at the Center for History and Philosophy of Physics, American Institute of Physics, New York.

[95] *Physikalische Abhandlungen und Vorträge*, vol. 3, p. 267.

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[96] E.g., cf. W. Heisenberg, *P*
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1959), p. 35.
[97] 56. *Jahrgang*, for 1900 (190
[98] A. L. Day, "Measurement o
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[99] Cambridge University Pre
[100] L. Holborn and S. Valenti
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turskala mit dem Stickstoffth
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(1907).
[101] W. W. Coblentz, "A chara
spectral energy curves," *Phys*
31, 314–319 (1910). Cf. also
"Versuche zur Prüfung des Wi
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35, 543–590 (1911).
[102] E. Warburg, G. Leith
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609–634 (1913); E. Warbur
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48, 410–432 (1915).
[103] W. Nernst and T. Wulf, "
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Berlin, intimated to him his feelings of having made a discovery comparable perhaps only to the discoveries of Newton.[96]

For the time being, at least until 1905, nobody in fact seems to have realized that Planck's was indeed "a discovery comparable perhaps only to the discoveries of Newton." Germany's official *Physical Abstracts* of that time, the *Fortschritte der Physik*, edited and published by the German Physical Society, mentioned Planck's contribution only in outline ("in äußersten Umrißen").[97] Outside Germany it seems to have attracted still less attention. An exception was Arthur L. Day's report on Planck's work to the 547th meeting of the Philosophical Society of Washington[98] in 1902. J. H. Jeans's first edition of his *Dynamical Theory of Gases*,[99] published in 1904, contained no reference whatever to Planck's law. In short, Planck's introduction of h seems to have been regarded at that time as an expedient methodological device of no deeper physical significance, although his radiation law was repeatedly subjected to experimental test. It was confirmed by Holborn and Valentiner,[100] by Coblentz,[101] and by Warburg and his collaborators.[102] On the other hand, as late as 1919 Nernst and Wulf[103] thought they had found deviations from Planck's law. Subsequent research, however, fully vindicated his result from both the experimental and the theoretical points of view.[104] With the increasing number of experimental

[96] E.g., cf. W. Heisenberg, *Physics and Philosophy* (G. Allen & Unwin, London, 1959), p. 35.

[97] 56. *Jahrgang*, for 1900 (1901), p. 338.

[98] A. L. Day, "Measurement of high temperature," *Science* (n.s.) *15*, 429–433 (1902).

[99] Cambridge University Press, 1904.

[100] L. Holborn and S. Valentiner, "Eine Vergleichung der optischen Temperaturskala mit dem Stickstoffthermometer bis 1600°," *Annalen der Physik 22*, 1–48 (1907).

[101] W. W. Coblentz, "A characteristic of spectral energy curves," *Physical Review 31*, 314–319 (1910). Cf. also E. Baisch, "Versuche zur Prüfung des Wien-Planckschen Strahlungsgesetzes im Bereich kurzer Wellenlängen," *Annalen der Physik 35*, 543–590 (1911).

[102] E. Warburg, G. Leithäuser, E. Hupka, and C. Müller, "Über die Konstante c des Wien-Planckschen Strahlungsgesetzes," *Annalen der Physik 40*, 609–634 (1913); E. Warburg and C. Müller, "Über die Konstante c des Wien-Planckschen Strahlungsgesetzes," *ibid. 48*, 410–432 (1915).

[103] W. Nernst and T. Wulf, "Über eine Modifikation der Planckschen Strahlungsformel auf experimenteller Grundlage," *Verhandlungen der Deutschen Physikalischen Gesellschaft 21*, 294–337 (1919). They proposed to add on the right-hand side of (1.16) a factor $(1 + \alpha)$, where α is a function of ν.

[104] H. Rubens and G. Michel, "Prüfung der Planckschen Strahlungsformel," *Physikalische Zeitschrift 22*, 569–577 (1921), confirmed Planck's formula by precision measurements and showed that the results obtained by Nernst and Wulf (see REF. 103) were erroneous. For subsequent theoretical derivations of Planck's formula cf. J. Weiss, "Über das Plancksche Strahlungsgesetz," *Physikalische Zeitschrift 10*, 193–195 (1909); P. Debye, "Der Wahrscheinlichkeitsbegriff in der Theorie der Strahlung," *Annalen der Physik 33*, 1427–1434 (1910); J. Larmor, "On the statistical and thermodynamical relations of radiant energy," *Proceedings of the Royal Society of London* (*A*), *83*, 82–95 (1910); W. Nernst, "Zur Theorie der spezifischen Wärme und über die Anwendung der Lehre von den Energiequanten auf physikalisch-chemische Fragen überhaupt," *Zeitschrift für Elektrochemie 17*, 265–275 (1911); P. Franck, "Zur Ableitung der Planckschen Strahlungsformel," *Physikalische Zeitschrift 13*, 506–507 (1912); A. Einstein and O. Stern, "Einige Argumente für die Annahme einer molekularen Agitation beim absoluten Nullpunkt," *Annalen der Physik 40*, 551–

confirmations of Planck's law, numerous attempts were made to evade Rayleigh's conclusion (1.9) without abandoning classical statistical mechanics and, in particular, the equipartition theorem.[105] The reason, as Lorentz put it, was undoubtedly that "we cannot say that the mechanism of the phenomena has been unveiled [by Planck's theory], and it must be admitted that it is difficult to see the reason for this partition of energy by finite portions, which are not even equal to each other, but vary from one resonator to the other."[106]

Another conceptual difficulty, which prevented the general acceptance of Planck's introduction of h, was undoubtedly the following fact. As shown by its dimension, this quantity represented an invariable unit of "action" (energy × time) or an "elementary quantum of action" ("elementares Wirkungsquantum"), as it was subsequently called. But it was clear that no principle of conservation of action exists in physics. It is therefore not surprising that the attempt to reconcile Planck's law with classical statistical mechanics was not abandoned even after Lorentz had shown that classical physics, that is, the equipartition theorem and Hamilton's principle, leads necessarily to Rayleigh's radiation law and its empirically untenable implications. As Lorentz put it, the ether is a system of infinitely many degrees of freedom, and the temperature of a ponderable body, in thermal equilibrium with it, on this assumption must necessarily be absolute zero, a result contrary to experience.

Lorentz made these statements in a series of lectures which he delivered in 1910 at the University of Göttingen. In this context the following historical comments are not without interest.

In 1908 the mathematician Paul Wolfskehl[107] of Darmstadt bequeathed the sum of 100,000 marks to the Academy of Sciences in Göttingen as an award for the first person to publish a complete proof of Fermat's famous *Last Theorem* (1637). In this theorem, it will be recalled, Fermat denied

560 (1913); M. Wolfke, "Zur Quantentheorie," *Verhandlungen der Deutschen Physikalischen Gesellschaft 15*, 1123, 1215 (1913); M. Wolfke, "Welche Strahlungsformel folgt aus der Annahme der Lichtatome?" *Physikalische Zeitschrift 15*, 308–310, 463 (1914); A. Einstein, "Zur Quantentheorie der Strahlung," *Mitteilungen der Physikalischen Gesellschaft, Zürich, 18*, 47–62 (1916), *Physikalische Zeitschrift 18*, 121–128 (1917); A. Rubinowicz, "Zur Quantelung der Hohlraumstrahlung," *ibid.*, 96–99 (1917); C. G. Darwin and R. H. Fowler, "On the partition of energy," *Philosophical Magazine 44*, 450–479, 823–842 (1922); C. G. Darwin and R. H. Fowler, "Partition functions for temperature radiation and the internal energy of a crystalline solid," *Proceedings of the Cambridge Philosophical Society 21*, 262–273 (1922); S. N. Bose, "Plancks Gesetz und Lichtquantenhypothese," *Zeitschrift für Physik 26*, 178–181 (1924); S. N. Bose, "Wärmegleichgewicht im Strahlungsfeld bei Anwesenheit von Materie, *ibid. 27*, 384–392 (1924); A. S. Eddington, "On the derivation of Planck's law from Einstein's equation," *Philosophical Magazine 50*, 803–808 (1925).

[105] H. A. Lorentz, "On the emission and absorption by metals of rays of heat of great wave-length," *Amsterdam Proceedings 1902–1903*, p. 666.

[106] H. A. Lorentz, *The Theory of Electrons* (1st ed. 1909, 2d ed. 1915; quoted from the Dover edition, New York, 1952), p. 80.

[107] "Bekanntmachung," *Göttinger Nachrichten 1908*, p. 103.

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[108] H. Poincaré, *Sechs Vorträ*
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[109] H. A. Lorentz, "Alte
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ɜr edition, New York, 1952), p. 80.
"Bekanntmachung," Göttinger Nach-
ɜn 1908, p. 103.

the existence of integers x, y, z, and n which satisfy $xyz \neq 0$, $n > 2$, $x^n + y^n = z^n$. It is also well known that the theorem has not yet been proved but has gained the unique distinction of being the problem for which the greatest number of incorrect "proofs" has ever been published.

What is not so well known, however, is the wise decision of the Wolfskehl committee to use the interest of the amount for the purpose of inviting prominent scientists as guest speakers to Göttingen. Such an invitation brought Poincaré there at the end of April, 1909. He gave six lectures on problems in pure and applied mathematics.[108] In his first talk (April 22) he spoke of Fredholm's equations in connection with the work of Hill and Helge von Koch, a subject whose relevance to quantum theory was not recognized until 1925; in his last lecture "La Mécanique Nouvelle" (April 28)—the only one he gave in French—he discussed the theory of relativity—incidentally, without mentioning the name of Einstein.

In the following year Lorentz was invited. From October 24 to 29, 1910, he delivered six lectures on "Old and New Problems in Physics"[109] which were subsequently edited by Born and published in the *Physikalische Zeitschrift*. The last three of these lectures dealt with the problem of black-body radiation. Three years later Sommerfeld spoke on problems of mathematical physics, and in the summer semester of 1914 Debye gave a series of lectures.

The last scientist to be invited on this program was Niels Bohr. His Göttingen lectures, delivered on June 12 to 22, 1922, had, as we shall see in due course, a decisive influence upon Pauli and Heisenberg. Bohr's "Seven Lectures on the Theory of Atomic Structure"[110] began with a general survey of atomic theory (first lecture), dealt with the correspondence principle and the adiabatic principle (second lecture), their applications (third lecture), discussed polyelectronic systems (fourth lecture), the periodic system (fifth lecture), x-rays and atomic structure (sixth lecture), and concluded with remarks on problems still to be solved. The subjects covered in these lectures were essentially the same as contained in Bohr's paper on the structure of atoms, published at that time.[111]

Mathematical research so far seems to have profited very little from Wolfkehl's incitement, and since the inflation in Germany depreciated the prize and in view of the historic impact of Bohr's lectures upon Pauli and Heisenberg it is perhaps no exaggeration to say that quantum theory

[108] H. Poincaré, *Sechs Vorträge aus der Reinen Mathematik und Mathematischen Physik* (Teubner, Leipzig, Berlin, 1910).

[109] H. A. Lorentz, "Alte und neue Fragen der Physik," *Physikalische Zeitschrift 11*, 1234–1257 (1910).

[110] A manuscript with notes on these lectures which Bohr delivered in German under the title "Sieben Vorträge über die Theorie des Atombaus" is found in the *Bohr Archive* under the title "Optegnelser til Forelaesningerne i Göttingen," Bohr Mss. No. 10.

[111] N. Bohr, "Der Bau der Atome und die physikalischen und chemischen Eigenschaften der Elemente," *Zeitschrift für Physik 9*, 1–67 (1922).

was the main beneficiary of the Wolfskehl Prize. Whether this statement will have to be modified in view of the recently proposed revival of the prize remains to be seen.

In concluding our discussion on the early development of the conception of energy quanta, in which Planck's derivation of the radiation law played the dominant role, we think it necessary to stress the following critical remarks.

As we have pointed out, Planck's derivation consisted of two separate parts: (1) a derivation of the relation (1.6) between the radiative energy density u_ν and the oscillator energy U,

$$u_\nu = \frac{8\pi}{c^3}\nu^2 U \tag{1.6}$$

a formula which Planck obtained by using exclusively the principles of classical electrodynamics (as shown in Appendix A); (2) a statistical treatment of the interaction among oscillators of different proper frequencies which resulted in the formula (1.15),

$$U = \frac{h\nu}{\exp\,(h\nu/kT) - 1} \tag{1.15}$$

By combining (1.6) and (1.15) Planck obtained his radiation law (1.16). We have also emphasized that these conclusions were adduced by Planck in order to provide a logical justification of his far-reaching interpolation mentioned above.

Planck's reasoning was inconsistent, however, as Einstein, in 1906, was the first to recognize.[112] For although either part of Planck's derivation of (1.16) was in itself consistent, their combination was logically incompatible. The reason was this: in the electrodynamical part (1) formula (1.6) is based on Maxwell's theory (see Appendix A) and the assumption that the oscillator energy is a continuously variable quantity, whereas in the statistical part (2) this same energy is treated as a discrete quantity, capable of assuming only values which are multiples of $h\nu$.

Referring to this inconsistency, Einstein remarked that "if the energy of a resonator can change only discontinuously, the usual theory of electricity cannot be applied for the calculation of the average energy of such a resonator in a radiation field. Planck's theory has, therefore, to assume that, although Maxwell's theory of elementary resonators is not applicable, the average energy of such a resonator, surrounded by radiation, is equal to that which would result from the calculation on the basis of Maxwell's theory of electricity."

"Such an assumption," continued Einstein, "would be plausible

[112] A. Einstein, "Zur Theorie der Lichterzeugung und Lichtabsorption," *Annalen der Physik 20*, 199–206 (1906).

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provided $\epsilon = h\nu$ were small throughout the observable spectrum compared to the average energy U of the resonator; but this is not the case."[113]

Three and a half years later, at the 81st meeting of the German Association of Scientists, held at Salzburg in September, 1909, Einstein[114] spoke on the development of our ideas on the nature and constitution of radiation. Repeating on this occasion his challenge to Planck's reasoning and resuming the question of whether the two parts cannot be reconciled with each other, Einstein pointed out that the previously mentioned condition, namely, that the energy quantum $\epsilon = h\nu$ be small in comparison with U, is certainly not satisfied. "A simple calculation shows," he declared, "that ϵ/U for $\nu = 0.5\ \mu$ and $T = 1700°\text{K}$ is not only not small compared to unity, but very large. It is approximately 6.5×10^7."

For Einstein this inconsistency was no reason to reject Planck's quantum theory as such. Having meanwhile proposed his ideas concerning light quanta, as we shall see in the next paragraph, Einstein saw in this inconsistency an indication that the foundations of the traditional radiation theory, based on Maxwell's electromagnetic theory, had to be revised.

The logical incompatibility of the two parts in Planck's derivation of his radiation law was a matter of great concern also for Peter Debye.[115] But contrary to Einstein, who hoped to overcome the difficulty by modifying the Maxwellian interaction between resonator and field, Debye attempted to resolve the inconsistency by eliminating altogether the role of ponderable resonators in Planck's derivation. Referring to the Jeans-Rayleigh computation of the number $N\,d\nu$ of vibrations in an enclosure of unit volume and frequency interval $d\nu$,

$$N\,d\nu = \frac{8\pi\nu^2}{c^3}\,d\nu$$

Debye assumed that the $N\,d\nu$ vibrations consist of $f(\nu)$ quanta of energy content $h\nu$ each, so that

$$u_\nu\,d\nu = \frac{8\pi h\nu^3}{c^3} f(\nu)\,d\nu$$

Defining "black radiation" as the most "probable radiation," that is, as the state with the greatest possible number of distributions of the $f(\nu)$ quanta among the $N\,d\nu$ receptors, Debye proved by using Planck's combinatorial formula that in this case $f(\nu) = [\exp(h\nu/kT) - 1]^{-1}$, a result which in combination with the preceding formula implied Planck's law of radiation. Debye thus showed that Planck's law and its implications

[113] *Ibid.*, p. 203.

[114] A. Einstein, "Über die Entwicklung unserer Anschauungen über das Wesen und die Konstitution der Strahlung," *Physikalische Zeitschrift 10*, 817–825 (1909).

[115] P. Debye, "Der Wahrscheinlichkeitsbegriff in der Theorie der Strahlung," *Annalen der Physik 33*, 1427–1434 (1910).

follow from the assumption alone that the energy as such is quantized in units of $h\nu$ and no knowledge concerning the properties of resonators or their mechanism is needed for this purpose. Debye's[116] assumption may be referred to as the "weak quantum postulate," in contrast to Planck's "quantum postulate," according to which also the energy content of an oscillator is always a multiple of $h\nu$.

1.3 The Concept of Quanta of Radiation

In the development of quantum theory discussed so far, the concept of energy elements or quanta had been regarded as applicable only to the mechanism regulating the interaction between matter and radiation: it was the material oscillator of frequency ν which could emit or absorb energy only in multiples of $h\nu$.

Meanwhile, however, an important conceptual development took place which led to a certain generalization of the conception of quanta. It began in 1905, when the general validity of the electromagnetic theory of light was seriously called into question by Einstein's article "On a heuristic viewpoint concerning the production and transformation of light."[117] In its importance for the future development of theoretical physics this essay may be compared with Einstein's classic paper on special relativity with which it appeared—together also with his famous study on Brownian motion—in the same volume of the *Annalen der Physik*. Although commonly referred to as Einstein's paper on the photoelectric effect, it discussed a problem of much wider significance and contained a suggestion which challenged classical physics perhaps to the same extent as did Planck's historic paper of 1900.

Einstein considered monochromatic radiation of frequency ν and of small density within the range of ν/T where Wien's radiation law (1.5) is valid. If v is the volume of the enclosure and $u(\nu)$ the spectral distribution function, the entropy could be expressed by the equation $S = v\int_0^\infty \varphi(u,\nu)\, d\nu$, where φ is a function of u and of ν. In order to find the explicit dependence of φ on u and ν, Einstein had two equations to start with: $\delta\int\varphi\, d\nu = 0$, expressing the fact that the entropy for the equi-

[116] Strictly speaking, to attain a stationary state of radiation (i.e., of maximum entropy) Debye needed the property of ponderable bodies to exchange radiation from one wavelength to another. He therefore defined his postulate of elementary quanta as follows: "Schwingungsenergie kann von ponderabelen Körpern aufgenommen werden und eventuell in Energie von anderer Schwingungszahl übergeführt werden nur in Form von Quanten von der Größe $h\nu$." *Ibid.*, p. 1430.

[117] Albert Einstein, "Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt," *Annalen der Physik 17*, 132–148 (1905). Recently translated into English by A. B. Arons and M. B. Peppard, *American Journal of Physics 33*, 367–374 (1965).

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librium state of the cavity radiation is a maximum, and $\delta\int u\,d\nu = 0$, expressing the conservation of energy. Introducing an undetermined multiplier, he obtained, for every choice of δu as a function of ν, the equation

$$\int \left(\frac{\partial\varphi}{\partial u} - \lambda\right) \delta u\,d\nu = 0$$

where λ and consequently also $\partial\varphi/\partial u$ are independent of ν. Taking $v = 1$, Einstein calculated the increase of entropy for dT as

$$dS = \int_{\nu=0}^{\infty} \frac{\partial\varphi}{\partial u}\,du\,d\nu$$

or, in view of the independence just proved, $dS = (\partial\varphi/\partial u)\,dE$, where dE is the heat added reversibly and hence subject to $dS = dE/T$. Comparison of the last two equations showed that $\partial\varphi/\partial u = 1/T$ irrespective of the particular form of $u(\nu)$. Solving Eq. (1.5) for T^{-1}, Einstein obtained the differential equation

$$\frac{\partial\varphi}{\partial u} = -(\beta\nu)^{-1}\log\frac{u}{\alpha\nu^3}$$

and after integration

$$\varphi(u,\nu) = -\frac{u}{\beta\nu}\left(\log\frac{u}{\alpha\nu^3} - 1\right)$$

The entropy of the radiation within the interval from ν to $\nu + d\nu$ and with the energy $E_\nu = vu_\nu\,d\nu$ was therefore given by the expression

$$S = -\frac{E_\nu}{\beta\nu}\left[\log\left(\frac{E_\nu}{\alpha v\nu^3\,d\nu}\right) - 1\right]$$

If the radiation, originally in volume v_0, is assumed to occupy volume v, the last equation shows that the change in entropy is

$$S - S_0 = \frac{E_\nu}{\beta\nu}\log\frac{v}{v_0}$$

or equivalently

$$S - S_0 = \frac{R}{N}\log\left(\frac{v}{v_0}\right)^{NE_\nu/\beta\nu R} \tag{1.17}$$

where N is Avogadro's number and R the gas constant. On the other hand, from the kinetic theory of gases, Einstein argued, it is well known, that the probability of finding n particles at an arbitrary instant of time within a partial volume v of the volume v_0 in which they were originally moving is given by $(v/v_0)^n$. Hence

$$S - S_0 = \frac{R}{N} \log \left(\frac{v}{v_0}\right)^n \tag{1.18}$$

Guided by the identity of the mathematical structure of (1.17) and (1.18), Einstein concluded that $E_\nu = n(R\beta\nu/N)$ and declared that with respect to the theory of heat, "monochromatic radiation of small density (within the range of validity of Wien's radiation law) behaves as if it consisted of independent energy quanta of magnitude $R\beta\nu/N$."[118] Wien's exponential coefficient β, expressed by Planck's constants, was of course h/k, as Planck's law readily showed for $h\nu \gg kT$ and $R/N = k$. In effect, therefore, Einstein stated that radiation behaved as if it were composed of a finite number of localized energy quanta $h\nu$ or "photons," as they were later called after G. N. Lewis[119] introduced this term in 1926.

The idea of a discontinuous distribution of radiant energy in space was, of course, completely at variance with the prevailing undulatory electromagnetic theory of light. Furthermore Einstein's suggestion of a granular structure of radiation seemed to counter one of the most well-founded and indisputable results of physical research. Was not the discovery of diffraction, first reported by Leonardo da Vinci,[120] rediscovered and investigated by Grimaldi,[121] and accountable only in terms of the wave theory of Huygens[122] and Young,[123] as Fresnel[124] has so masterly

[118] "Monochromatische Strahlung von geringer Dichte (innerhalb des Gültigkeitsbereiches der Wienschen Strahlungsformel) verhält sich in wärmetheoretischer Beziehung so, wie wenn sie aus voneinander unabhängigen Energiequanten von der Größe $R\beta\nu/N$ bestünde." *Ibid.*, p. 143.

[119] Lewis thought it inappropriate to speak of a "quantum of light," "if we are to assume that it spends only a minute fraction of its existence as a carrier of radiant energy, while the rest of the time it remains an important structural element within the atom. . . . I therefore take the liberty of proposing for this hypothetical new atom, which is not light but plays an essential part in every process of radiation, the name photon." G. N. Lewis, "The conservation of photons," *Nature 118*, 874–875 (1926).

[120] G. Libri, *Histoire des Sciences Mathématiques en Italie* (J. Renouard, Paris, 1838–1841), vol. 3, p. 54.

[121] Francesco Maria Grimaldi, *Physicomathesis de lumine, coloribis et iride aliisque adnexis libri duo* (Benatii, Bologna, 1665).

[122] Christiaan Huygens, *Traité de la Lumière* (P. Van der Aa, Leiden, 1960); *Treatise on Light*, translated by S. P. Thompson (Macmillan, London, 1912).

[123] Thomas Young, "On the theory of light and colour," *Philosophical Transactions of the Royal Society of London 92*, 12–24 (1802); *A Course of Lectures on Natural Philosophy and the Mechanical Arts* (J. Johnson, London, 1807), especially Lecture 39, vol. 1, pp. 457–471.

[124] Augustin Jean Fresnel, "Sur la diffraction de la lumière, où l'on examine particulièrement le phénomène des fringes colorées que présentent les ombres des corps éclairés par un point lumineux," *Annales de Chimie et de Physique 1*, 239–281 (1816); *Oeuvres Complètes d'Augustin Fresnel* (Imprimerie Impériale, Paris, 1866), vol. 1, pp. 89–122, 129–170; "Mémoire sur la diffraction de la lumière," *Mémoires de l'Académie des Sciences 1819; Annales de Chimie et de Physique 11*, 246–296, 377–378 (1819); *Oeuvres*, vol. 1, pp. 247–384; English translation in *The Wave Theory of Light*, edited by H. Crew (American Book Company, New York, 1900), pp. 79–144; German translation in *Abhandlungen über die Beugung des Lichts*, translated by F. Ritter, *Ostwald's Klassiker der exakten Wissenschaften No. 215*, (Akademische Verlagsgesellschaft, Leipzig, 1926).

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[125] Sir Isaac Newton, *Ph*
Naturalis Principia Mathemati
Societatis regiae, London, 1687
section 14, propositions 94–98;
a Treatise of the Reflexions, R
Inflexions and Colours of Light
London, 1704), book 1, part 1, p
6, theorem 5.

[126] Pierre-Simon Laplace, *Tra*
canique Céleste (Duprat, Paris, 1
4, p. 241; *Exposition du Système*
(Courcier, Paris, 1813), 4th ed

[127] Jean Baptiste Biot, *Traité d*
Expérimentale et Mathématique (
Paris, 1816), vols. 3 and 4.

[128] Armand Hippolyte Fizeau,
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79, 167–169 (1850).

[129] Jean Bernard Léon Fouca
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[130] H. Fizeau and L. Breguet,

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shown, an incontestable deathblow to any corpuscular conception of light such as proposed by Newton,[125] Laplace,[126] or Biot?[127] Have not Fizeau,[128] Foucault,[129] and Breguet,[130] following a suggestion by Arago,[131] established without doubt that the velocity of light is less in water than in air and thus brought forth a crucial[132] decision of the "particle-versus-wave" issue in favor[133] of the latter? As Hanson[134] recently pointed out, these experiments proved the undulatory nature of light but certainly did not prove that light cannot also be corpuscular. For classical physics has only gradually shaped the notions of particle and wave to logical contraries or opposites. In fact, a thorough scholarly study of the history of the logical relationship between these two notions, so important for modern physics, and on their development to the status of fundamentally incompatible and mutually exclusive conceptions still remains a project for future research. In any case, when Young declared, "It is allowed on all sides, that light either consists in the emission of very minute particles from luminous substances, which are actually projected, and continue to move, with the velocity commonly attributed to light, or in the excitation of an undulatory motion, analogous to that which constitutes sound, in a highly light and elastic medium pervading the universe; but the judgments of philosophers of all ages have been much divided with respect to the preference of one or the other of these opinions,"[135] he obviously used the connective "or" in the dis-

[125] Sir Isaac Newton, *Philosophiae Naturalis Principia Mathematica* (jussu Societatis regiae, London, 1687), book 1, section 14, propositions 94–98; *Opticks, or a Treatise of the Reflexions, Refractions, Inflexions and Colours of Light* (S. Smith, London, 1704), book 1, part 1, proposition 6, theorem 5.

[126] Pierre-Simon Laplace, *Traité de Mécanique Céleste* (Duprat, Paris, 1808), vol. 4, p. 241; *Exposition du Système du Monde* (Courcier, Paris, 1813), 4th ed., p. 327.

[127] Jean Baptiste Biot, *Traité de Physique Expérimentale et Mathématique* (Deterville, Paris, 1816), vols. 3 and 4.

[128] Armand Hippolyte Fizeau, "Sur une expérience relative à la vitesse de propagation de la lumière," *Comptes Rendus 29,* 90–92 (1849); "Versuch, die Fortpflanzungsgeschwindigkeit des Lichts zu bestimmen," *Poggendorff's Annalen der Physik 79,* 167–169 (1850).

[129] Jean Bernard Léon Foucault, "Méthode générale pour mesurer la vitesse de la lumière dans l'air et les milieux transparents," *Comptes Rendus 30,* 551–560 (1850); "Allgemeine Methode zur Messung der Geschwindigkeit des Lichts in Luft und durchsichtigen Mitteln," *Poggendorff's Annalen der Physik 81,* 434–442 (1850).

[130] H. Fizeau and L. Breguet, "Note sur l'expérience relative à la vitesse comparative de la lumière dans l'air et dans l'eau," *Comptes Rendus 30,* 562–563, 771–774 (1850); "Notiz in Betreff eines Versuchs über die comparative Geschwindigkeit des Lichts in Luft und in Wasser," *Poggendorff's Annalen der Physik 81,* 442–444 (1850), *82,* 124–127 (1851).

[131] Dominique François Jean Arago, "Sur un système d'expérience à l'aide duquel la théorie de l'émission et celle des ondes seront soumises à des épreuves décisives," *Comptes Rendus 7,* 954–965 (1838); "Über ein System von Versuchen, mit Hülfe dessen die Emissions- und die Undulationstheorie auf entscheidende Proben gestellt werden können," *Poggendorff's Annalen der Physik 46,* 28–41 (1839).

[132] On the "cruciality" of such experiments see N. R. Hanson, *The Concept of the Positron* (Cambridge University Press, 1963), pp. 18–24.

[133] "Hiermit haben sich Fizeau und Foucault das Verdienst erworben, die Emissionstheorie endgiltig widerlegt zu haben." A. Winkelmann, *Handbuch der Physik* (E. Trewendt, Breslau, 1894), p. 10.

[134] REF. 132, p. 12.

[135] REF. 123, Lecture 39, p. 457.

junctive sense of the Latin *aut* (and not *vel*). Arago[136] even considered the issue as a mathematically or logically unequivocal dichotomy. Yet, for the physics of the later nineteenth century the spatial distribution of energy was either discrete, as in the corpuscular-kinetic theory of Newtonian mechanics, or continuous, as in Maxwell's electromagnetic theory, but never both discrete and continuous for one and the same category of physical phenomena.

Strictly speaking, as Einstein[137] once pointed out, the success of the wave theory of light was the first breach in Newtonian physics, for corpuscular-kinetic conceptions were replaced by field-theoretic notions. But throughout the later nineteenth century these two schemes of conceptions enjoyed a rather peaceful coexistence.

Even Einstein, in the beginning of the paper under discussion,[138] admitted that the classical theory of light based on continuous space functions was so firmly established that it would probably never be replaced by another theory. But, he continued to say, optical observations take account only of time averages, and it is quite conceivable that such a theory of light, in spite of its convincing verifications by experiments on interference and diffraction, may prove itself insufficient whenever instantaneous values of those functions have to be considered or whenever interactions of matter with radiation, as in the processes of emission and absorption, are involved. Einstein, it seems, did not know that similar doubts had previously been raised by J. J. Thomson. Faced by difficulties in explaining quantitatively the ionization caused by Röntgen rays, as x-rays were still called at that time, Thomson declared in 1903: "If, for example, we consider a plane at right angles to the direction of propagation of the rays the energy is not distributed uniformly over this plane, but the distribution of energy has as it were a structure, although an exceedingly fine one, places where the energy is large alternating with places where it is small, like mortar and bricks in a wall."[139] The effect which led Thomson to his conjecture of "patches of energy," namely, photoionization, was one of the instances, together with Stokes' law and the photoelectric effect, for

136 "Je me propose de montrer dans cette Note, comment il est possible de décider, sans équivoque, si la lumière se compose de petites particules *émanant* des corps rayonnants, ainsi que le voulait *Newton*, ainsi que l'ont admis la pluspart des géomètres modernes; ou bien si elle est simplement le résultat des *ondulations* d'un milieu très rare et très élastique, que les physiciens sont convenus d'appeler l'Ether. Le système d'expériences que je vais décrire, ne permettra plus, ce me semble, d'hésiter entre les deux théories rivales. Il tranchera *mathématiquement* (j'emploie à dessein cette expression); il tranchera mathématiquement une des questions les plus débattues de la philosophie naturelle." REF. 131, p. 954.

137 "The theoretical system, built up by Newton with his powerful and logical intellect, should have been overthrown precisely by a theory of light . . . by the Huygens-Young-Fresnel wave theory of light." A. Einstein, "The new field theory," *The Times* (London), No. 45,118, Feb. 4, 19 9, pp. 13–14.

138 REF. 117.

139 J. J. Thomson, *Conduction of Electricity through Gases* (Cambridge University Press, 1903), p. 258.

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nius,[142] in the course of thei
ently of Hertz and at about
the same effect, although
cause of the phenomenon.
as to the mechanism by w
recognizing that much more
task. In fact, his discovery
Wiedemann and Ebert[143] c
electric arc discharges the n
one. Hallwachs,[144] also in 18
freshly polished zinc plates t

140 "Nach den Resultaten uns
suche hat das ultraviolette I
Fähigkeit, die Schlagweite der I
gen eines Inductoriums und ve
Entladungen zu vergrößern." I
"Über einen Einfluß des ultra
Lichtes auf die electrische En
Wiedemannsche Annalen der P
982–1000 (1887).

141 A. Schuster, "Experiment
discharge of electricity throug
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