

$$|C_n(t)|^2 = 2\pi t \Delta_t (E_n - E_i) |M|^2,$$

- where

$$\lim_{t \rightarrow \infty} \Delta_t (E_n - E_i) = \delta(E_n - E_i)$$

and

$$M = \langle n | H_1 | i \rangle + \sum_k \frac{\langle n | H_1 | k \rangle \langle k | H_1 | i \rangle}{E_i - E_k}$$

where  $k$  labels intermediate states.  $k$  is supposed to run over a complete set of states of both fields, but most terms are zero.

To see what can occur in 2nd order theory (since in this application  $\langle n | H_1 | i \rangle = 0$ ) we look at the general structure of the

- product of matrix elements (with  $H_1 = H_T$ ):

$$\langle n | H_1 | k \rangle \langle k | H_1 | i \rangle \approx \sim$$

$$\int d^3\vec{x} d^3\vec{x}' \langle n | (b^\dagger d)(a a^\dagger)(b d^\dagger) | k \rangle \langle k | (b^\dagger d)(a a^\dagger)(b d^\dagger) | i \rangle$$

There are  $2^6 = 64$  combinations of creation/annihil. ops, but most terms are zero (for the given  $|i\rangle$  and  $|n\rangle$ ). First, since we must create one photon on each application of  $H_T$ , the  $a$  terms do not contribute (only the  $a^\dagger$  terms). (Recall that time advances from right to left, initial to final states). Next, since we must annihilate  $e^- e^+$  in two steps, in which there appear four

$b, b^+, d, d^+$  operators, we must use 3 annihil. and 1 creation operators. Moreover the pattern must be AAAC or AACA, because if we use ACAA (annihilating both initial  $e^+e^-$  in the first step) then the intermediate state has no  $e^+$  or  $e^-$  for the annihilation operator to act on in the second step. Thus we find the two possibilities in the table below:

Case	$\bar{\Psi}(\vec{x})$	$A(\vec{x})$	$\psi(\vec{x})$	$\bar{\Psi}(\vec{x}')$	$\bar{A}(\vec{x}')$	$\psi(\vec{x}')$
1a	$d_{p's'}$	$a_{\lambda'}^+$	$b_{p's}$	$b_{p's}^+$	$a_{\lambda}^+$	$b_{ps}$
2a	$d_{p's_2}$	$a_{\lambda}^+$	$b_{ps}$	$d_{p's'}$	$a_{\lambda'}^+$	$d_{p's_2}^+$

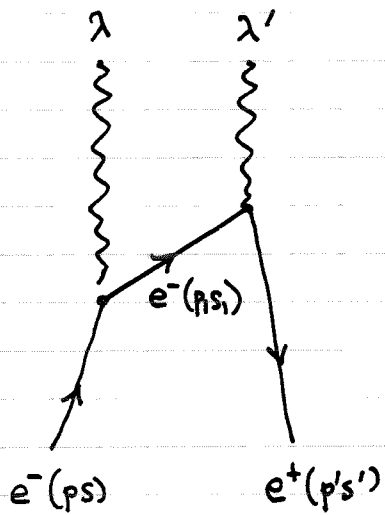
Two more possibilities are created by swapping  $\lambda \leftrightarrow \lambda'$ :

1b	$d_{p's'}$	$a_{\lambda}^+$	$b_{p's}$	$b_{p's}^+$	$a_{\lambda'}^+$	$b_{ps}$
2b	$d_{p's_2}$	$a_{\lambda'}^+$	$b_{ps}$	$d_{p's'}$	$a_{\lambda}^+$	$d_{p's_2}^+$

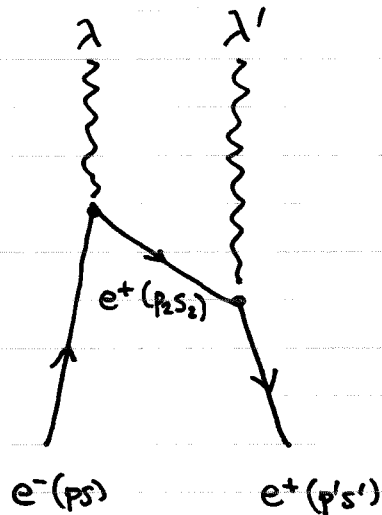
Only 4 of the 64 terms are nonzero.

Notice that in cases 2a, 2b the  $d d^+$  operators are not normal ordered. We must swap them, introducing a minus sign, to carry out the normal ordering. The minus sign is important, because amplitudes are added before being squared.

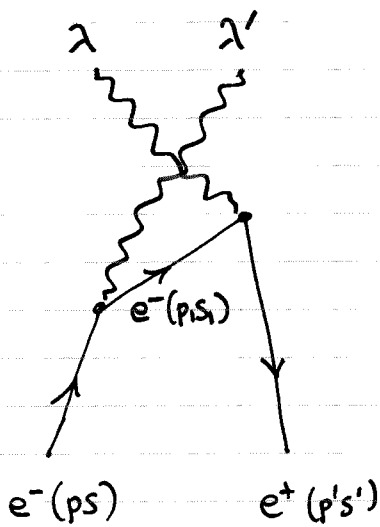
These cases have the Feynman diagrams:



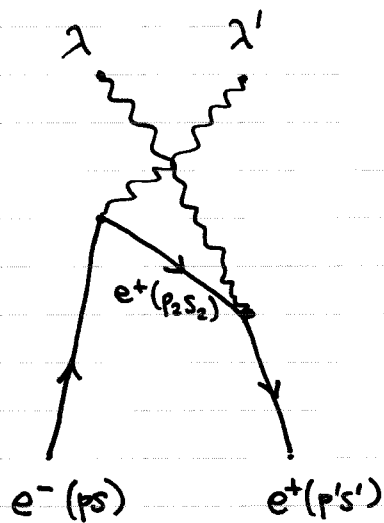
(1a)



(2a)



(1b)



(2b)

In cases 1ab we have an electron in the intermediate state, in cases 2ab a positron.

We are using "old fashioned" perturbation theory. It is based on nonrelativistic ideas and is not covariant. It gives correct

answers for relativistic calculations, but is not as elegant or simple as covariant perturbation theory. The latter is described by Sakurai, but we will not have time to go into it. However, in covariant perturbation theory diagrams 1a and 2a are combined into a single diagram. These two diagrams differ only in the order in which the initial electron and positron are annihilated. In covariant perturbation theory there are only two diagrams.

Instead of wending our way through creation and annihilation operators to go from the initial state to the final state, we can glue the elementary Feynman diagrams on pp. 6-7 together to create the diagrams on p. 10. We must be careful about the normal ordering of operators and sign changes, however. The compound diagrams must take us from the <sup>given</sup> initial to the final states.

Now we can compute the contributions to the transition amplitude coming from the four Feynman diagrams. We need to translate the general notation

$$M = \sum_k \frac{\langle n | H_1 | k \rangle \langle k | H_1 | i \rangle}{E_i - E_k} \quad (\text{1st order term vanishes})$$

into the notation for our problem. The initial and final energies are

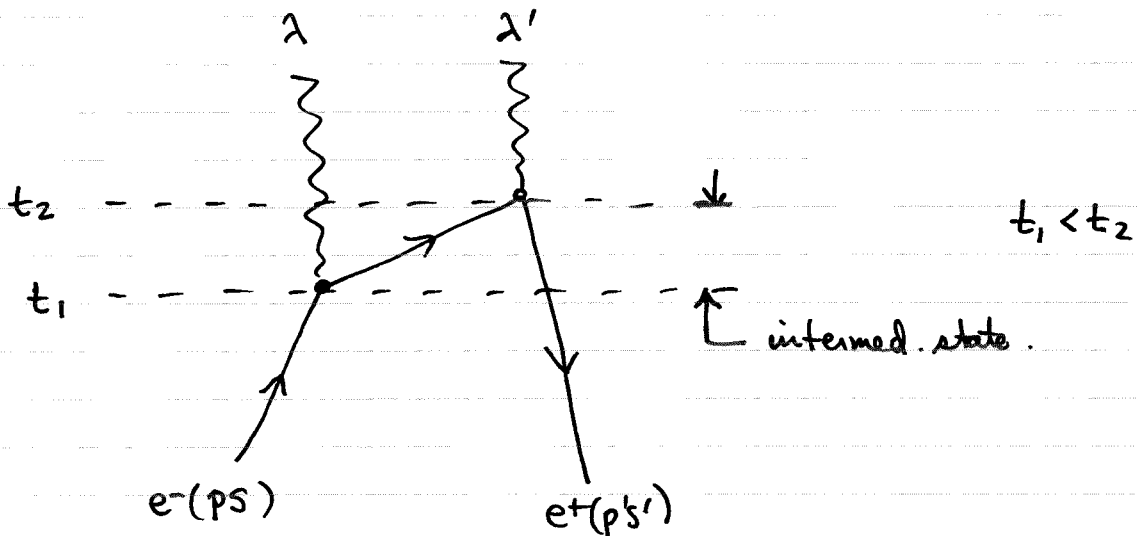
$$E_i = E + E' \quad (\text{initial } e^+, e^- \text{ energy})$$

$$E_n = \omega + \omega' \quad (\text{final photon energies})$$

Begin with case 1a. Do the energy denom. first. The energy of the intermediate state is

$$E_k = \omega + E_1 + E' \quad (\text{final photon} + \text{intermed. electron} + \text{initial positron})$$

as you can see from the Feynman diagram:



~~Next do the matrix~~ so here

$$E_i - E_k = E - \omega - E_1$$

Now do the matrix elements, starting with  $\langle k | H_1 | i \rangle$

(proceeding in time sequence). Here the intermediate state is

$$|k\rangle = a_\lambda^\dagger b_{p_1 s_1}^\dagger d_{p' s'}^\dagger |0\rangle$$

the initial state is  $|i\rangle = b_{p s}^\dagger d_{p' s'}^\dagger |0\rangle$ , so for the field part of the matrix element we get

$$\begin{aligned} \langle k | b_{p_1 s_1}^\dagger a_\lambda^\dagger b_{p s} | i \rangle \\ = \langle 0 | d_{p' s'} b_{p_1 s_1} a_\lambda b_{p s}^\dagger a_\lambda^\dagger b_{p s} b_{p s}^\dagger d_{p' s'}^\dagger | 0 \rangle = 1. \end{aligned}$$

(The  $a$ 's commute with the  $b$ 's and  $d$ 's, which anticommute w. each other.)

The rest of the matrix element is

$$\begin{aligned} \langle k | H_1 | i \rangle = e \int d^3 \vec{x} \frac{1}{V} \sqrt{\frac{2\pi}{V}} \sqrt{\frac{n^2}{\epsilon \epsilon_0 \omega}} (\bar{u}_{p_1 s_1} \vec{\gamma} u_{p s}) \cdot \hat{\epsilon}^* \\ \times e^{i \vec{x} \cdot (-\vec{p}_1 - \vec{k} + \vec{p})} \end{aligned}$$

Look first at  $\vec{\gamma} \cdot \hat{\epsilon}^*$ . (Here we write  $\hat{\epsilon} \equiv \hat{\epsilon}_\lambda$ ,  $\hat{\epsilon}' \equiv \hat{\epsilon}_{\lambda'}$ .)

Obviously  $\gamma^\mu = (\gamma^0, \vec{\gamma})$ . Define a polarization 4-vector by

$$\epsilon^\mu = (0, \hat{\epsilon}).$$

This is like what we did in defining a spin 4-vector for electrons,

we went to the rest frame and set  $s^\mu = (0, \hat{s})$ , then  $s^\mu$  in

any other frame follows by Lorentz transforming. But the photon