Physics 221B
Spring 2012
Homework 22
Due Monday, April 2, 2012

Reading Assignment:

Handwritten notes for the week on the Dirac equation. On p. 21 of the notes posted on Wednesday, March 14, there is an error: there are two places where the expression $2g^{\alpha\beta}$ occurs, where it should read $g^{\alpha\beta}$ (without the 2). The error makes no difference in the final result, since it's still a constant, that commutes with γ^μ.

My lectures on the introduction to the Dirac equation are mostly following the presentation of Bjorken and Drell. See the reprints, chapter 2, pp. 16–21 for some on this week's lectures.

Sakurai covers similar territory, but in a notation different from what I'm using and different from Bjorken and Drell, and in a different order. But he makes a number of interesting and important comments, so please look at *Advanced Quantum Mechanics*, pp. 84–85, 89–91 and 95–99.

I am making the homework due on the Monday after spring break.

1. Continuation of the problem from last week.

(d) Using the Dirac-Pauli representation, write the 2-component Dirac spinor as

$$\psi = e^{-imc^2t/\hbar} \begin{pmatrix} \phi \\ \chi \end{pmatrix}, \tag{2}$$

and assume that the energy is $E = mc^2 + \text{small}$. Find an approximation giving χ in terms of ϕ, and use it to obtain an effective Schrödinger equation for the upper component ϕ. Is the resulting Schrödinger equation realistic for any problems in our real (3-dimensional) world?

(e) Assume that the 2-component Dirac wave function transforms under proper Lorentz transformations Λ in $2+1$ dimensions according to

$$\psi'(x) = D(\Lambda)\psi(\Lambda^{-1}x), \tag{3}$$

where $D(\Lambda)$ is some (as yet unknown) 2×2 representation of the proper Lorentz transformations in $2+1$ dimensions and $x = (ct, x_1, x_2)$ (1,2 mean x, y). Assuming that $\psi(x)$

satisfies the free particle Dirac equation, and that $\psi'(x)$ is given by Eq. (3), demand that $\psi'(x)$ also satisfy the free particle Dirac equation and thereby derive a condition that the representation $D(\Lambda)$ must satisfy.

(f) Write out explicitly the matrices $D(\Lambda)$ for the case of pure rotations and pure boosts. Do this in the Dirac-Pauli representation. Show that if you work in the Maiorana representation, the D-matrices are purely real.