$\boxed{21.11 \quad p.\ 468}$

$$ds^2 = \frac{1}{y^2}\left(dx^2 + dy^2\right)$$

$$g_{\mu\nu} = \begin{pmatrix} 1/y^2 & 0 \\ 0 & 1/y^2 \end{pmatrix}, \qquad g^{\mu\nu} = \begin{pmatrix} y^2 & 0 \\ 0 & y^2 \end{pmatrix}$$

Seems easiest to get $\Gamma^{\mu}_{\alpha\beta}$ from the general formula, instead of "by hand." Let

$$\Gamma_{\sigma\alpha\beta} = \frac{1}{2}\left(\partial_\beta g_{\sigma\alpha} + \partial_\alpha g_{\sigma\beta} - \partial_\sigma g_{\alpha\beta}\right),$$

so that $\Gamma^{\mu}_{\alpha\beta} = g^{\mu\sigma}\,\Gamma_{\sigma\alpha\beta}$. Only $\frac{\partial}{\partial y}$ is nonzero.

$$\Gamma_{xxx} = 0 \ ; \qquad \Gamma_{xxy} = \Gamma_{xyx} = \frac{1}{2}\frac{\partial}{\partial y} g_{xx} = -\frac{1}{y^3}$$

$$\Gamma_{xyy} = \frac{1}{2}\left(\frac{\partial g_{xy}}{\partial y} + \frac{\partial g_{xy}}{\partial y}\right) = 0$$

$$\Gamma_{yxx} = \frac{1}{2}\left(-\frac{\partial g_{xx}}{\partial y}\right) = \frac{1}{y^3}$$

$$\Gamma_{yxy} = \frac{1}{2}\left(\frac{\partial g_{xy}}{\partial y} - \frac{\partial g_{xy}}{\partial y}\right) = 0 = \Gamma_{yyx}$$

$$\Gamma_{yyy} = \frac{1}{2}\,\frac{\partial g_{yy}}{\partial y} = -\frac{1}{y^3}.$$

So, nonvanishing $\Gamma^{\mu}_{\alpha\beta}$ are:

$$\boxed{\begin{aligned} \Gamma^{x}_{xy} &= \Gamma^{x}_{yx} = -\frac{1}{y} \\ \Gamma^{y}_{xx} &= \frac{1}{y} \\ \Gamma^{y}_{yy} &= -\frac{1}{y} \end{aligned}}$$

$$R^{\mu}{}_{\cdot\nu\alpha\beta} = \partial_\alpha \Gamma^{\mu}_{\beta\nu} + \Gamma^{\sigma}_{\beta\nu}\Gamma^{\mu}_{\sigma\alpha} - (\alpha \leftrightarrow \beta)$$

This is a 2D problem, so there is only one indep. component of R, which we take to be $R^{x}{}_{\cdot yxy}$. (see prob. 21.8.)

$$R^{x}{}_{\cdot yxy} = \partial_x \Gamma^{x}_{yy} + \Gamma^{\sigma}_{yy}\Gamma^{x}_{\sigma x} - \partial_y \Gamma^{x}_{xy} - \Gamma^{\sigma}_{xy}\Gamma^{x}_{\sigma y}$$

$$= 0 + \Gamma^{y}_{yy}\Gamma^{x}_{yx} - \partial_y \Gamma^{x}_{xy} - \Gamma^{x}_{xy}\Gamma^{x}_{xy}$$

$$= 0 + \left(-\frac{1}{y}\right)\left(-\frac{1}{y}\right) - \partial_y\left(-\frac{1}{y}\right) - \left(-\frac{1}{y}\right)^2$$

$$= \frac{1}{y^2} - \frac{1}{y^2} - \frac{1}{y^2} = \boxed{-\frac{1}{y^2} = R^{x}{}_{\cdot yxy}}$$

Now get Ricci ~~tensor~~:

$$R_{xx} = R^{\mu}{}_{\cdot x\mu x} = R^{x}{}_{\cdot xxx} + R^{y}{}_{\cdot xyx} = R_{x\cdot}{}^{y}{}_{xy} = R^{x}{}_{\cdot yxy} = -\frac{1}{y^2}$$

$$R_{xy} = R^{\mu}{}_{\cdot x\mu y} = R^{x}{}_{\cdot xxy} + R^{y}{}_{\cdot xyy} = 0$$

$$R_{yy} = R^{\mu}{}_{\cdot y\mu y} = R^{x}{}_{\cdot yxy} + R^{y}{}_{\cdot yyy} = -\frac{1}{y^2}$$

$$R_{\mu\nu} = \begin{pmatrix} -1/y^2 & 0 \\ 0 & -1/y^2 \end{pmatrix} \qquad R^{\mu}{}_{\cdot\mu} = \mathrm{tr}\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = -2.$$

$$\boxed{R = -2}$$