Physics 139
Spring 2014
Homework 12
Due Friday, April 25, 2014

Reading Assignment: Please read Hartle, pp. 434–441; 445-450.

1. Problem 20.15, p. 443.

2. Problem 20.16, p. 443.

3. Problem 20.18, p. 443.

4. If $\mathbf{V}$ is a vector with components V^μ, then $\nabla_{\mathbf{D}}\mathbf{V}$ is another vector, the directional derivative of $\mathbf{V}$ along $\mathbf{D}$, with components $(\nabla_{\mathbf{D}}\mathbf{V})^\mu = D^\alpha \, \nabla_\alpha V^\mu$, where

$$\nabla_\alpha V^\mu = \frac{\partial V^\mu}{\partial x^\alpha} + \Gamma^\mu_{\alpha\beta} \, V^\beta. \tag{1}$$

Equivalently, we can say that $\nabla\mathbf{V}$ is a tensor, the *covariant derivative* of $\mathbf{V}$, with components $\nabla_\alpha V^\mu$. Find an expression for $\nabla_\beta \nabla_\alpha V^\mu$, the components of the second covariant derivative of $\mathbf{V}$. Express

$$\nabla_\alpha \nabla_\beta V^\mu - \nabla_\beta \nabla_\alpha V^\mu \tag{2}$$

in terms of the components of $\mathbf{V}$ and the Riemann tensor,

$$R^\mu{}_{\nu\alpha\beta} = \frac{\partial \Gamma^\mu_{\beta\nu}}{\partial x^\alpha} - \frac{\partial \Gamma^\mu_{\alpha\nu}}{\partial x^\beta} + \Gamma^\mu_{\alpha\sigma} \, \Gamma^\sigma_{\beta\nu} - \Gamma^\mu_{\beta\sigma} \, \Gamma^\sigma_{\alpha\nu}. \tag{3}$$

The point is that the covariant derivative along different directions does not commute, and the commutator involves the Riemann curvature tensor.

5. Problem 21.2, p. 466.