Problem 1

A long straight wire carries a steady current $I$ along the $z$ axis, in the $+z$ direction. We define points $a = (d, 0, 0)$ and $b = (0, d, 2d)$, where $d$ is a positive distance parameter. For a straight-line path from $a$ to $b$, calculate the following. [You can integrate along a different path if you can explain why the answer should be the same.]

$$\int_a^b \mathbf{B} \cdot d\mathbf{l}$$

$$\int_a^b \mathbf{B} \times d\mathbf{l}$$

Problem 2

We define points $c = (d, 0, 0)$ and $d = (0, d, 0)$, where $d$ is a positive distance parameter. A current $I$ flows around a triangular loop, from the origin to $c$, then to $d$ and then back to the origin.

(a) Find the exact value of the vector potential at the point $(x, 0, 0)$, assuming $x > d$. Use the conventions of Eq. (5.66).

(b) Calculate the dipole moment of the loop.

(c) Use the leading term in the multipole expansion, Eq. (5.85), to calculate the vector potential at the point $(x, 0, 0)$.

(d) Show that to leading order, your answers to parts (a) and (c) agree.

Problem 3

In this problem, we will solve a modified form of Problem 5.31(c). [Please do not hand in solutions to 5.31 (a) and (b)]. Let $\mathbf{F} = y^2 \hat{x} + z^2 \hat{y} + x^2 \hat{z}$. Calculate $\mathbf{A}$ and $\nabla \times \mathbf{A}$. Change $\mathbf{A}$ by the gradient of a scalar, so it is divergenceless.
Problem 4

This is a continuation of Problem 4 on last week’s homework assignment. We still have the compass on the horizontal table, but we remove the current-carrying wire. The Cartesian coordinate system and the value for the Earth’s magnetic field $B_{Earth}$ are the same as before.

A small permanent magnet is now brought in from the right (positive $x$ axis) from a great distance. We will treat this magnet as a point magnetic dipole $m$. The poles are not labeled. It is known that $m$ points in either the $+x$ or $-x$ direction.

We assume the compass is in a uniform magnetic field, the sum of $B_{Earth}$ and an approximately uniform magnetic field equal to the magnetic field produced by $m$ at the center of the compass.

When $m$ is at $x = 40 \text{ cm}$, the compass needle points about 10 degrees west of north, that is, $\alpha = 10^\circ$. At $x = 30 \text{ cm}$, we have $\alpha = 20^\circ$. At $x = 25 \text{ cm}$, we have $\alpha = 35^\circ$. At $x = 20 \text{ cm}$, we have $\alpha = 60^\circ$.

(a) Explain this behavior using the formula for the magnetic field of a point dipole. Find an exponent $n$ and a trigonometric function ($\sin$, $\cos$, etc.) such that when you plot the trig function of $\alpha$ versus $x^n$ you would expect a straight line. Plot the actual data in this way and comment on how straight the line is. What could cause a departure from straight-line behavior here?

(b) Determine the direction of $m$, and estimate its magnitude.