Problem 1

Conductor #1 is a sphere of radius \( R = 0.10 \) m and is centered at the origin of a Cartesian coordinate system. Conductor #2 has an unusual shape located above Conductor #1. Though the shape is unusual, this conductor possesses a rotational symmetry about the \( z \)-axis. There is a gap between the conductors. This capacitor is charged to a potential difference of \( V = 600 \) V. The charge on Conductor #1 is \(+Q\) and the charge on Conductor #2 is \(-Q\).

The magnitude of the electric field at the north pole of the sphere is \( E_N = 5.0 \) kV/m. The magnitude of the electric field at the south pole of the sphere is \( E_S = 2.5 \) kV/m. At both poles, the direction of the field is radially outward.

Assuming the form \( \sigma_0 + \sigma_1 \cos(\theta) \) for the surface charge density on the sphere, find the constants \( \sigma_0 \) and \( \sigma_1 \) and the total charge on the sphere. Compute the capacitance. For each of these questions, derive a formula and then plug in the numbers.

Problem 2

A point charge \( q \) is located at \((0,0,L)\) in a standard Cartesian coordinate system.

(a) Write down the potential \( V(x, y, z) \).

(b) We are interested in the behavior of the potential near the origin. By computing derivatives up to second order at the origin, or by other means, find an approximation to the potential as a second-order polynomial in \( x, y \) and \( z \).

(c) Verify that this polynomial satisfied Laplace’s equation.

Problem 3

Using the answer given in problem 3.40 as a starting point, please answer the following questions.

(a) What is the surface charge density on the cylinder?

(b) Integrating this surface charge density over the right half of the cylinder (the half of the cylinder that is closer to the \(+\lambda\) wire), what is the resulting linear charge density? If the integral is too difficult, you may approximate the surface charge density as \( \sigma_0 + \sigma_1 \cos(\phi) \) and proceed that way (Explain the values you used for \( \sigma_0 \) and \( \sigma_1 \)).
Problem 4

Using the solution to Problem 2.52 (see last week’s homework), find the capacitance per unit length of the following two long conducting cylindrical shells. The outer cylindrical shell has radius $2R$ and is centered on the $x$-axis. The inner cylindrical shell has radius $R$ and is centered on a line parallel to the $x$-axis, given by $y = y_0$ and $z = 0$. The value of $y_0$ is such that there is no contact between the cylindrical shells. (You may wish to first study Problem 2.43).

Problem 5

In this problem, we solve a modified form of Problem 3.7 (page 129). Rather than being located at $(0, 0, 3d)$, the charge $+q$ is located at $(2d, 2d, 3d)$. Find the $x$, $y$ and $z$ components of the force on the $+q$ charge.

Problem 6

We define a “P4 sphere” to be a sphere like the one defined in Problem 4 of last week’s homework. Let $W_{P4}$ denote the energy required to assemble the sphere, as computed in last week’s homework. We place one P4 sphere at the origin and another at the point $(0, 0, L)$, where $L > 2R$.

(a) Using the principle of superposition, find the total potential $V(r)$ at all points in space. There are three different regions to consider as explained in Problem 3 of last week’s homework.

(b) Use the formula $W = \frac{1}{2} \int \rho V d\tau$ to find the energy required to assemble this configuration.

(c) What is the force between two P4 spheres whose centers are a distance $r$ apart?

(d) Imagine two P4 spheres assembled a great distance apart. Compute the work required to bring them closer together so that their centers are separated by a distance $L$. This work plus $2W_{P4}$ should equal your answer to part (b).

Further Problems

Solve or study published solutions to the following problems. Please DO NOT HAND IN your solutions to these problems. The purpose is to make sure you understand the solutions by the end of the week. We can discuss these problems in office hours or discussion sections.

Problem 3.40

Problem 3.41