Today

I Last time

II Relativistic effects from Lorentz transformations

III Units where $c = 1.$

\[
\begin{pmatrix}
  c t' \\
  x' \\
  y' \\
  z'
\end{pmatrix} = \begin{pmatrix}
  \cosh \theta & -\sinh \theta & 0 & 0 \\
  -\sinh \theta & \cosh \theta & 0 & 0 \\
  0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
  c t \\
  x \\
  y \\
  z
\end{pmatrix}
\]

or

\[
c t' = x (ct - \frac{y}{c} x) \quad \text{where} \quad \frac{y}{c} = \tanh \theta
\]

\[
x' = x - \frac{y}{c} ct
\]

\[
y' = y
\]

\[
z' = z
\]

II (i) As we have seen the spacetime diagram depicting a boosted frame is:

Note that $\Delta t' = t'_B - t'_A = 0,$ hence
using the inverse lorentz transformation (which we easily obtain by taking \( t' \rightarrow t, x' \rightarrow x, \) etc and \( \gamma \rightarrow -\gamma \))

\[
\Delta t = \gamma (\Delta t' + \frac{\Delta x'}{c}) = \gamma \frac{\Delta x'}{c}
\]

This is precisely the relativity of simultaneity — two events separated by \( \Delta x' \) and such that \( \Delta t' = 0 \) have non-zero time separation, \( \Delta t \to 0 \) of time, but the latter depends on frame.

\[\begin{align*}
\Delta t &= \gamma (\Delta t' + \frac{\Delta x'}{c}) \\
&= \gamma \frac{\Delta x'}{c}
\end{align*}\]

\[\text{The invariant interval gives}\]

\[L^2 = c^2 \Delta t^2 + L_x^2\]

But, from the Lorentz transformation,

\[t' = 0 \Rightarrow t = \frac{V}{c^2} x \Rightarrow \gamma \Delta t = \frac{\Delta x}{c} \]

so that

\[\begin{align*}
L^2 &= L_x^2 \left(1 - \frac{V^2}{c^2}\right) \\
&= L_x^2 \left(1 - \frac{1}{\gamma^2}\right)
\end{align*}\]

or

\[L = L_x / \gamma, \text{ precisely length contraction}\]
(iii) Velocity addition: Suppose two frames are in relative motion with the second moving at speed \( v \)

\[
\begin{align*}
\mathbf{S} & \quad \mathbf{S}' \\
\mathbf{x} & \quad \mathbf{x}' \\
\mathbf{d}t & \quad \mathbf{d}t'
\end{align*}
\]

and that there is a particle moving through space, so that in \( \mathbf{S} \): \( x(t), y(t), z(t) \) and in \( \mathbf{S}' \): \( x'(t'), y(t'), z(t') \)

precisely the Einstein velocity addition rule. In most intros to SR we don’t look at \( V_y' \), so let’s do it:

\[
V_y' = \frac{dy'}{dt'} = \frac{dy}{dt} \left( 1 - \frac{v}{c} \frac{dx}{dt} \right)
\]

\[
= \frac{dy}{dt} \left( \frac{V_y}{\sqrt{1 - \frac{v^2}{c^2}}} \right)
\]

The “transverse” components \( V_x' \) and \( V_z' \) transform in a richer way.

provide descriptions of the \( \frac{73}{3} \) particle motion. Then by Lorentz trans:

\[
V_x' \overset{\text{def.}}{=} \frac{dx'}{dt'} = \frac{y(c \, dt - \frac{v}{c} \, dx)}{\sqrt{1 - \frac{v^2}{c^2} \, dx}}
\]

Dividing numerator and denominator by \( dt \) we obtain,

\[
V_x' = \frac{y \left( \frac{dx}{dt} - v \right)}{\sqrt{1 - \frac{v^2}{c^2} \, dx}}
\]

\[
= \frac{V_x - v}{\sqrt{1 - \frac{V_x^2}{c^2}}}
\]

(Invoking)

III Because all observers agree on the speed of light we can use it as a scale for measuring speeds. From now on when we say

\( v = \frac{1}{3} \), \( \rightarrow \) \( v = \frac{1}{3} c \)

That is, we’ll use units where \( c = 1 \). In these units we also have

\( t = 3 \) meters \( \rightarrow t = \frac{3 \, m}{c} = 10^{-8} \) s.

See Hartle’s Appendix A for a nice discussion.