Today

I What is G.R.?

II Logistical Interlude

III Principal of Equivalence

Come to class!

Ask questions!!

I'm indebted to my teachers:

David Griffiths, Robert Littlejohn, Carlo Rovelli

What is gravity?

(b) Einstein's theory of gravity

(i) Newtonian Gravity

Universal gravitation: \( F = -\frac{G m_1 m_2}{r^2} \)

Second law: \( F = m_1 \ddot{a} \)

\( m_2 \ddot{a} = -\frac{G m_1 m_2}{r^2} \)

But this law is inconsistent with Special Relativity (S.R.)

General Relativity

Day 1

I What is G.R.?

(a) Generalized Special Relativity

Ok to work in any reference (ref. frame moving)

(At constant velocity)

Leads to myriad wonderful questions:

What is the nature of time? space? What does it mean to make an observation? What is energy?

Contrast Elec. & Mag. — already relativistic. => Why not modify gravity just as we did with Coulomb's law

\( F = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{r^2} \)

(It doesn't work!) Here's why:

Inspiration: do the law of universal gravitation what Maxwell, Ampere, Faraday did
for Coulomb's law (e.g. $g \rightarrow m$).

[doesn't work]

Sources of E & M fields:
electric charge & current

$p, \vec{j} \rightarrow J^\mu = (cp, \vec{j})$

While the fields $\vec{E}$ & $\vec{B}$ form a
tensor

$$F^{\mu\nu} = \begin{pmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & B_z & -B_y \\ -E_y/c & -B_z & 0 & B_x \\ -E_z/c & B_y & -B_x & 0 \end{pmatrix}$$

kinetic, potential, B2 rest) via
E = mc². So, maybe all forms of
energy are sources of gravity.

Ok. But, energy is not a
Lorentz Scalar — it's one component
of the energy-momentum 4-vector:

$$p^\mu = (E/c, \vec{p})$$

So, momentum will also be a source
of gravity.

These are connected through $p^{\mu}/3$
Maxwell's equations:

$$\frac{\partial F^{\mu\nu}}{\partial x^\nu} = \mu_0 J^\mu$$

Why not substitute $g$ (or $g \epsilon$)
$\rightarrow m$ (or $g_{m}$)? Because mass,
unlike charge, is not additive
— total mass of composite
structure reflects all forms
of energy contained (e.g.

Moreover what we need is
energy momentum density—they fit together to make the
(energy-momentum-) stress tensor

$$\tau^{\mu\nu} = \begin{pmatrix} \text{En.} \\
\text{dens.} \\
\text{momentum}
\text{density} \\
\text{pressure, etc.} \end{pmatrix}$$
So we're looking for an equation of the form:

\[ ??? = k T^{\mu\nu} \]

a constant containing Newton's constant \( G \).

It took Einstein 12 years to find "?" s.

III Principal of Equivalence

Recall Newton: \( \sum \vec{F} = -G \frac{M^2}{r^2} \)

So, mass plays two unrelated roles — on the left it is inertia (i.e. a measure of \( 1/\|\vec{a}\| \)), and on the right the strength of gravity — and cancels out.

II Logistical Interlude

(i) Schedule 4th class hour
(ii) Schedule office hours

Results:
(i) Th 8-9pm
(ii) Th 9-10pm and Fri 10-11am.

All objects fall under gravity with the same acceleration.

Inertial mass = grav. mass

This leads into the idea that gravity is geometry.