Today's Outline:

I Survey
II Last lecture/Where are we?
III Collections of particles
IV Why are we distinguishing $\omega$ and $\Gamma$?

V Announce Midterm 2: Oct 28th (Fri.)
   • Covers Chs 7, 8, 9
   • Bring blue book
   • One pg. single sided of notes

Lecture 21
October 17th, 2011

I survey

We recently passed the half way mark for the course and I would like your feedback. I will use this, as best I can, to improve this course and to improve my teaching going forward.

I appreciate both constructive critique and positive feedback.

1. What percentage of the time do you feel you’ve understood the main point of a lecture when it ended?

2. Is the lecture moving too fast, too slow or at the right pace?

3. What would you change were you teaching the course? What’s going well?

4. In a few weeks we will have completed the core foundation of the course. I currently plan to cover: Hamiltonian Mechanics, Chaos and Collision theory.

Pick your top 3 of the following topics (or pick 2 and say which you would like to spend more time with):

- Hamiltonian theory, chaos, collision theory, continuum mechanics, Hamilton-Jacobi theory, or computational techniques: symplectic integrators
Any additional comments:

II Last Lecture/Where are we?
We investigated rotating frames for two reasons:
1. To understand motion in noninertial frames; modify Newton’s 2nd law to include fictitious forces.
2. To set up context for investigating rotating bodies (as opposed to frames).

This formula is amazing (!): we can treat collections of particles as a single pt. particle with mass $M$ subject to $\vec{F}_{\text{ext}}$.

Consider a rigid body and define $\vec{r}'_\alpha$ as the position of $m_\alpha$ w.r.t. the C.M.

III Collections of particles

CM: $\vec{R} = \frac{1}{M} \sum_{\alpha} m_\alpha \vec{r}'_\alpha = \frac{1}{M} \sum_{\alpha} m_\alpha \dot{\vec{r}}'_\alpha$ ($\alpha = 1, \ldots, N$)

Einstein summation convention: repeated index means sum over that index.

Total Momentum:
$\vec{P} = \sum_{\alpha} p_{\alpha} = m_\alpha \vec{v}'_\alpha = M \vec{R}$

(\text{since} $\frac{d}{dt}(m_\alpha \vec{v}'_\alpha) = \frac{d}{dt}(M \vec{R})$)

Total external Force:
$\vec{F} = \vec{F}_{\text{ext}} = M \dot{\vec{R}}$

Aside:
Throughout this chapter, Taylor forgoes integration unless it is absolutely necessary. A nice convention but do be careful. I

$\vec{r}'_\alpha = \vec{R} + \vec{r}'_\alpha$

Avg. mom. $\vec{L}$:
$\vec{L} = \vec{r}'_\alpha \times \vec{p}'_\alpha = \vec{r}'_\alpha \times M \dot{\vec{r}}'_\alpha$ (sum)

Tot. Avg. mom. $\vec{L} = \sum_{\alpha} \vec{L}_\alpha = \sum_{\alpha} \vec{r}'_\alpha \times M \dot{\vec{r}}'_\alpha$

This simplifies:

$= \sum_{\alpha} (\vec{R} + \vec{r}'_\alpha) \times m_\alpha (\dot{\vec{R}} + \dot{\vec{r}}'_\alpha)$
So,
\[
L = \sum \dot{R} \times m_0 \ddot{R} + \sum \dot{R} \times m_0 \dot{\mathbf{r}}_0 + \sum \dot{\mathbf{r}}_0 \times m_0 \dot{\mathbf{r}}_0
+ \sum \dot{\mathbf{r}}_0 \times m_0 \ddot{\mathbf{r}}_0
\]
where \( \dot{\mathbf{r}}_0 \) is the position relative to the CM,
\[
= \dot{R} \times m_0 \dot{R} + \dot{R} \times (\sum m_0 \dot{\mathbf{r}}_0) + (\sum m_0 \dot{\mathbf{r}}_0) \times \dot{R}
+ \sum m_0 \dot{\mathbf{r}}_0 \times m_0 \ddot{\mathbf{r}}_0
\]
\[
= \dot{R} \times \dot{R} + \sum m_0 \dot{\mathbf{r}}_0 \times m_0 \ddot{\mathbf{r}}_0
\]
\[
\Rightarrow L = L_{\text{motion of CM}} + L_{\text{motion relative to CM}}
\]

Also simplifies:
\[
\dot{\mathbf{r}}_0^2 = (\dot{\mathbf{r}}_0 + \dot{\mathbf{r}}')^2 = \dot{R}^2 + \dot{\mathbf{r}}'^2 + 2 \dot{R} \cdot \dot{\mathbf{r}}'
\]
\[
\Rightarrow T = \frac{1}{2} \sum m_0 \dot{\mathbf{r}}_0^2 + \frac{1}{2} \sum m_0 \dot{\mathbf{r}}'^2 + \dot{R} \cdot \dot{\mathbf{r}}'
\]
\[
= \frac{1}{2} M \dot{R}^2 + \frac{1}{2} m_0 \dot{\mathbf{r}}'^2
\]
In words
\[
T = T_{\text{motion of CM}} + T_{\text{rotation relative to CM}}
\]

For a rigid body any motion relative to the CM is a rotation (Euler's theorem)

For example a planet orbiting the sun (assume sun fixed):
\[
L = L_{\text{orbital angular momentum due to motion around the sun}}
+ L_{\text{spin angular momentum due to motion relative to CM}}
\]

Total kinetic:
\[
T = \frac{1}{2} \sum m_0 \dot{\mathbf{r}}_0^2
\]

Energy

and so
\[
T = T_{\text{motion of CM}} + T_{\text{rotation about CM}}
\]

Finally, if the forces on and within a body are conservative,
\[
U = U_{\text{ext}} + U_{\text{int}}
\]
gives rise to forces that hold body together.

Total external potential energy
\[
U_{\text{ext}} = \sum U_{\text{dist. between particles}} = \text{constant we can drop.}
\]

\[
U_{\text{int}} = \sum U_{\text{potential energy}}
\]