I Orbit transfer

Special case: tangential thrust at perigee.

Choose thrust so \( \theta = 0 \Rightarrow \phi_0 = 0, \)
\( e_1 = 0, e_2 = 0. \) In this case the orbit equation becomes

\[
\frac{r_1}{1 + e_1} = \frac{r_2}{1 + e_2} = \frac{c_1}{1 + e_1}.
\]

Thrust factor: \( e_2 = 2e_1 \) (\( \text{speed up} \))
\( e_2 = 2e_1 \) (\( \text{slow down} \))

Perigee: \( E = 0 \) and \( L = \mu v \)

\[
E_2 = 2E_1 \quad \text{and} \quad c_2 = 2c_1 \quad (C = \frac{\mu}{2})
\]

Example: Most efficient transfer to Mars:

"Hohmann transfer." Both the earth and Mars have roughly circular orbits:

<table>
<thead>
<tr>
<th>Planets</th>
<th>Eccentricity</th>
<th>Orbit Radius</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth</td>
<td>( E_e = 0.0167 )</td>
<td>( R_e = 1.5 \times 10^{11} ) m</td>
</tr>
<tr>
<td>Mars</td>
<td>( E_m = 0.0933 )</td>
<td>( R_m = 2.3 \times 10^{11} ) m</td>
</tr>
</tbody>
</table>

A Hohmann transfer involves two thrusts, the first takes you from the circular (Earth) orbit to an elliptical orbit.
that intersects the desired circular orbit (Mars in our case) and the second that brings you onto the target orbit. (Note: I'm speaking about orbits around the Sun now.)

Take \( \varepsilon_c \approx \varepsilon_t \approx 0 \). Denote the \( \frac{1}{2} \) transfer orbit quantities \( c_1 \) and \( c_t \). Then

\[
C_t = c_t = R_e \quad \text{(recall geometrical interp. of c)}
\]

and

\[
C_t = \lambda^2 R_e
\]

\[
E_t = \lambda^2 (0) + (\lambda^2 - 1) = \lambda^2 - 1
\]

Now \( R_M \) must be apogee \( A \)

Transfer \( 50 \),

\[
R_M = \frac{c_t}{1 - E_t} = \frac{\lambda^2 R_e}{1 - (\lambda^2 - 1)} = \frac{\lambda^2 R_e}{2 - \lambda}
\]

II. Non-inertial Frames

- Newton's laws only hold in inertial frames
- We are careful to write down the Lagrangian in an inertial frame.

For the next few lectures we explore non-inertial frames.

Need a 10% increase in speed! I leave it to you to find \( \lambda_t \), the thrust factor to go into Mars' orbit.

\[
=) 2R_m - \lambda^2 R_m = \lambda^2 R_e
\]

\[
=) \lambda^2 (R_e + R_m) = 2R_m
\]

\[
\lambda = \sqrt{\frac{2R_m}{R_e + R_m}} = \sqrt{\frac{4.6}{3.8}} \approx 1.10
\]
Consider the position of a particle. In the frame $S$, this is measured to be $\mathbf{r}_0$ and it obeys Newton's 2nd law:

$$m\ddot{\mathbf{r}}_0 = \mathbf{F}$$

In $S$, they measure $\mathbf{r}$ and $\dot{\mathbf{r}}$:

$$\ddot{\mathbf{r}}_0 = \dot{\mathbf{r}} + \dot{\mathbf{V}}$$

Particle's vel. = particles vel. w.r.t. ground + vel. of moving frame w.r.t. ground

This then implies:

$$\ddot{\mathbf{r}}_0 = \ddot{\mathbf{r}} + \dot{\mathbf{A}}$$

so that:

$$m\ddot{\mathbf{r}} = m\ddot{\mathbf{r}}_0 - m\dot{\mathbf{A}}$$

$$= \mathbf{F} - m\dot{\mathbf{A}}$$

This looks a lot like Newton's 2nd law. In fact, it is if we agree to introduce an "inertial force"

$$\mathbf{F}_{\text{inertial}} = -m\dot{\mathbf{A}}$$

Examples: Airplane takeoff, elevator, car, etc.

This is computationally fast, conceptually subtle route!

III Great Example: Tides

The Earth is a noninertial frame because it is rotating. We will discuss this more next lecture. However, it is also a noninertial frame because it is accelerating towards the Moon (sun also). The tides are a combined effect of the Moon's gravitational attraction...
of the ocean and of this acceleration.

Tidal bulge

The acceleration \( \ddot{A} \) is magnitude and direction are determined by treating the Earth and moon as point masses concentrated at their respective centers.

Forces on \( m \):
1. Earth's gravity: \( mg \)
2. Moon's gravity: \(-GM_m \frac{d}{d^2} \), \( M_m = \) moon's mass
3. Net non-gravitational force: \( \vec{F}_{ng} \)

And the earth is a noninertial frame with acceleration,

\[ \ddot{A} = -\frac{GM_m}{d^2} \frac{d}{d^2} \]

We can use our noninertial frame law

\[ m\ddot{x} = \dot{F} - mA \]

On the other hand, the gravitational pull of the moon on the oceans is greater on the side closer to the moon and weaker on the opposite side. This is why there are two high tides each day.

Illustrate direction of tidal force on figure.

Next time: Quantitative analysis of tides. Begin rotating frames.