Note: This homework is due in two weeks. There will be no class on Feb. 19.


1. Review the familiar theorem which says that the eigenvalues of a Hermitian operator are real and the eigenspaces corresponding to distinct eigenvalues are orthogonal. (It is in the lecture notes.)

An operator $A$ is said to be normal if it commutes with its Hermitian conjugate, $[A, A^\dagger] = 0$. Notice that Hermitian, anti-Hermitian, and unitary operators are all normal. (An anti-Hermitian operator satisfies $A^\dagger = -A$.) Prove that the eigenspaces of a normal operator corresponding to distinct eigenvalues are orthogonal. If you can’t do this, you will get partial credit for proving the result in the special case of unitary operators. Hint: you will have to use the postulates satisfied by the metric or scalar product, see the lecture notes. Also notice, the eigenvalues of a normal operator are not necessarily real.

2. The group $C_{3v}$ is the covering group (proper and improper) of the ammonia molecule. Imagine the hydrogen atoms at the vertices of an equilateral triangle in the $x$-$y$ plane, as in Fig. 4.1 in the book, with the nitrogen atom on the $z$-axis. The group $C_{3v}$ consists of six rotations, three proper and three improper. (Recall that $D_3$ consists of six proper rotations.) Call the three proper rotations $(E, R_1, R_2)$; these are the same as $(E, R_1, R_2)$ of group $D_3$ (see p. 7 of the lecture notes for 1/29/03). Let $\sigma_v$ be a reflection in the “vertical” $y$-$z$ plane, and define $R_3 = \sigma_v$, $R_4 = \sigma_v R_1$, $R_5 = \sigma_v R_2$.

Write out the six $3 \times 3$ matrices containing the components of the action of $C_{3v}$ on ordinary 3-dimensional space, with respect to the usual $(\hat{x}, \hat{y}, \hat{z})$ basis.

Now consider the induced transformations of these rotations on the set of functions, $(\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6) = (x^2, y^2, z^2, xy, yz, zx)$. Call the induced transformations $\tilde{R}_1$, etc. Write out the $6 \times 6$ matrices for the action of the induced transformations $\tilde{R}_1$, $\tilde{R}_4$ and $\tilde{R}_5$, and verify that $\tilde{R}_1 \tilde{R}_5 = \tilde{R}_4$.

In this problem, sign errors are serious errors.