Reading Assignment: Read pp. 12–17 of the book, and study lecture notes for Jan. 29. We didn’t cover too many pages of the book, but I went into some issues in lecture that are not in the book. In particular, I covered cosets and Lagrange’s theorem, which is relegated to problem 2.4 by the book.

1. This is basically problem 2.3. The group $D_4$ is the group of proper covering operations of a square. The square lies in the $x$-$y$ plane (the horizontal plane), centered at the origin and aligned with the $x$-$y$ axes. The square has one 4-fold axis of symmetry (the $z$-axis), and four 2-fold axes of symmetry. Let the rotations about the 4-fold axis be $R_1$, $R_2$, $R_3$, where

$$R_k = R(\hat{z}, k\pi/2), \quad k = 1, 2, 3, \quad (1)$$

and let the rotations about the 2-fold axes be

$$R_k = R(\hat{n}_k, \pi), \quad k = 4, 5, 6, 7, \quad (2)$$

where

$$\hat{n}_4 = \hat{x},$$

$$\hat{n}_5 = (\hat{x} + \hat{y})/\sqrt{2},$$

$$\hat{n}_6 = \hat{y},$$

$$\hat{n}_7 = (-\hat{x} + \hat{y})/\sqrt{2}. \quad (3)$$

(a) Construct the group table for $D_4$.

(b) Consider the cyclic subgroup of order 4. List its left cosets; its right cosets. Are they the same?

(c) Consider the 2-element subgroup, $\{E, R_4\}$. Construct its left cosets; its right cosets. Are they the same?
2. In class, after discussing Lagrange’s theorem, we defined the order of a group element \( g \in G \) as the smallest integer \( n \) such that \( g^n = e \), and we showed that this must be a divisor of the order of the group itself (because \( g \) generates a cyclic subgroup of order \( n \)). This part did not get into the notes. Use this fact to classify all distinct groups of order 4. Groups are considered distinct if they are not isomorphic. Call the group elements \( e, a, b, c \), and exhibit the group multiplication tables for each distinct group. Indicate which groups are Abelian and which are non-Abelian.