Physics 221B
Spring 2011
Homework 26
Due Friday, May 4, 2012

Reading Assignment:

Please read the lectures notes for the lectures of April 23 to April 27.

1. Please use natural units ($\hbar = c = 1$) in this problem. Positrons were first observed by Anderson in 1932 in cosmic ray tracks in a cloud chamber. The experimental apparatus consisted of a series of parallel plates of lead (a high Z material), separated by air gaps. The gamma rays were produced when high energy muons (the principal component of cosmic rays at the earth's surface) passed close to a nucleus. These gamma rays, in turn, when passing close to another nucleus further below, produced electron-positron pairs.

When we discussed Mott scattering in class, we adopted a model in which the second quantized electron-positron field interacts with an external potential $\Phi(\mathbf{x})$ (such as $\Phi = Ze/r$ produced by a nucleus), which is treated as a given c-number field. In this model, the interaction Hamiltonian is

$$H_1 = \int d^3\mathbf{x} \, : J^\mu(\mathbf{x}) A_\mu(\mathbf{x}) :, \tag{1}$$

where J^μ is the electron field operator,

$$J^\mu(\mathbf{x}) = -e\bar{\psi}(\mathbf{x})\gamma^\mu\psi(\mathbf{x}), \tag{2}$$

and where

$$A^\mu = A^\mu_{\text{ext}} = (\Phi, 0). \tag{3}$$

If we wish to allow the emission and absorption of photons (real or virtual), we must include the quantized electromagnetic field. Therefore we write

$$A^\mu = A^\mu_{\text{qu}} + A^\mu_{\text{ext}}, \tag{4}$$

where A^μ_{qu} is the transverse, quantized field operator in the Coulomb gauge, $A^\mu_{\text{qu}} = (0, \mathbf{A})$. Here $\mathbf{A}$ is given by Notes Eq. 40.20. (But use natural units for this problem.) We must also include the instantaneous Coulomb interaction in the Hamiltonian,

$$H_{1,\text{Coul}} = \frac{1}{2} \int d^3\mathbf{x} \, d^3\mathbf{x}' \, \frac{: \rho(\mathbf{x})\rho(\mathbf{x}') :}{|\mathbf{x} - \mathbf{x}'|}, \tag{5}$$

where

$$\rho(\mathbf{x}) = -e\psi(\mathbf{x})^\dagger\psi(\mathbf{x}). \tag{6}$$

Thus, the overall interaction Hamiltonian is

$$H_1 = H_{1,\text{ext}} + H_{1,\text{qu}} + H_{1,\text{Coul}}. \tag{7}$$

(a) Consider a process in which a photon passes close to a nucleus, and an electron-positron pair is produced. (This process cannot happen in free space because of energy-momentum conservation.) Let (ps) be the 4-momentum and spin of the outgoing electron, let $(p's')$ be the 4-momentum and spin of the outgoing positron, let $\lambda = (\mathbf{k}, \boldsymbol{\epsilon})$ be the mode of the incident photon. Also write

$$\begin{aligned} p^\mu &= (E, \mathbf{p}), \\ p'^\mu &= (E', \mathbf{p}'), \\ k^\mu &= (\omega, \mathbf{k}). \end{aligned} \tag{8}$$

As discussed in the notes, through second order, time-dependent perturbation theory gives an effective transition probability as

$$P = 2\pi t \sum_n \Delta_t(E_n - E_i)|M|^2, \tag{9}$$

where

$$M = \langle n|H_1|i\rangle + \sum_k \frac{\langle n|H_1|k\rangle\langle k|H_1|i\rangle}{E_i - E_k}. \tag{10}$$

Here we use a general notation, where n is a variable final state that we sum over, and k is an intermediate state.

Find and draw all Feynman diagrams which contribute to this process at lowest order in $\alpha = e^2$. Count the external potential Φ as containing one power of e (since we are thinking of $\Phi = Ze/r$). Indicate the interaction with the nucleus by an X drawn next to an electron or positron line, as we did with Mott scattering.

(b) Pick out two Feynman diagrams which differ from one another only in the time ordering of the creation of the outgoing electron and positron. Work out in detail the contribution to M from each of these Feynman diagrams, and write down your answers. Express your answer in terms of the Fourier transform of the potential, as we did with Mott scattering,

$$\tilde{\Phi}(\mathbf{q}) = \frac{1}{(2\pi)^{3/2}} \int d^3\mathbf{x}\, e^{-i\mathbf{q}\cdot\mathbf{x}}\, \Phi(\mathbf{x}). \tag{11}$$

Define all symbols you use (unless they are obvious, like e). In this part, a sign error is a serious error.

(c) Combine the two terms, and express the result in terms of the Feynman electron propagator,

$$D_F(p) = \frac{\not{p} + m}{p^2 - m^2}. \tag{12}$$

Unfortunately, to carry this problem farther involves too much calculation for a homework problemx, but the steps remaining to obtain the cross section are straightforward.