Physics 221B
Spring 2011
Homework 28
Due Friday, April 29, 2011 at 5pm

Reading Assignment: Reprint on the Foldy-Wouthuysen transformation (chapter 4 of Bjorken and Drell); handwritten lecture notes on Foldy-Wouthuysen transformation; lecture notes for 4/20/11 and 4/22/11; Sakurai, *Advanced Quantum Mechanics*, pp. 131–145.

1. This is Bjorken and Drell problem 4.2, with the steps laid out in more detail. Do this problem in the following way. First, use natural units, $\hbar = c = 1$. Next, take the modified Dirac equation to be

$$\left(\not{p} - q\not{A} - \frac{\kappa e}{4m}\sigma_{\mu\nu}F^{\mu\nu} - m\right)\psi = 0, \tag{1}$$

where m is the mass, q the charge, and κ the strength of the anomalous magnetic moment term. For the electron, $q = -e$ and $\kappa = 0$; for the proton, $q = e$ and $\kappa = 1.79$; and for the neutron, $q = 0$ and $\kappa = -1.91$. For consistency, use the convention of Bjorken and Drell for $F_{\mu\nu}$,

$$F_{\mu\nu} = \frac{\partial A_\mu}{\partial x^\nu} - \frac{\partial A_\nu}{\partial x^\mu}, \tag{2}$$

even though this is opposite Jackson's convention.

(a) Write out the modified Dirac Hamiltonian, and show that it is Hermitian.

(b) Show that probability is conserved, i.e.,

$$\frac{\partial J^\mu}{\partial x^\mu} = 0, \tag{3}$$

where J^μ is defined exactly as for the unmodified Dirac equation, $J^\mu = \bar{\psi}\gamma^\mu\psi$.

(c) Covariance. Suppose $\psi(x)$ satisfies the modified Dirac equation (1), and let

$$\begin{aligned} \psi'(x) &= D(\Lambda)\psi(\Lambda^{-1}x), \\ A'^\mu(x) &= \Lambda^\mu{}_\nu\, A^\nu(\Lambda^{-1}x), \\ F'^{\mu\nu}(x) &= \Lambda^\mu{}_\alpha\, \Lambda^\nu{}_\beta\, F^{\alpha\beta}(\Lambda^{-1}x). \end{aligned} \tag{4}$$

Then show that $\psi'(x)$ satisfies the modified Dirac equation (1), but with Lorentz transformed fields $A'^\mu(x)$ and $F'^{\mu\nu}(x)$ instead of the original fields.

(d) Assume $\mathbf{E} = 0$, $\mathbf{B} \neq 0$ (in order to see what the effective magnetic moment of the particle is). Perform a simple nonrelativistic approximation as in pp. 2–6 of the lecture notes for March 30, and show that you get the right g-factors for the proton and neutron.

(e) Carry out a systematic Foldy-Wouthuysen transformation for the neutron as requested by problem 4.2. Remember $q = 0$, which simplifies the calculation. Order the terms in powers of $v/c = \eta$, as done in class, and carry the expansion out to order η^4.