Reading Assignment: Reprint on the Foldy-Wouthuysen transformation (chapter 4 of Bjorken and Drell); handwritten lecture notes on Foldy-Wouthuysen transformation; lecture notes for 4/20/11 and 4/22/11; Sakurai, *Advanced Quantum Mechanics*, pp. 131–145.

1. This is Bjorken and Drell problem 4.2, with the steps laid out in more detail. Do this problem in the following way. First, use natural units, \( \hbar = c = 1 \). Next, take the modified Dirac equation to be

\[
\left( \not{p} - q \not{A} - \frac{\kappa e}{4m} \sigma_{\mu \nu} F^{\mu \nu} - m \right) \psi = 0,
\]

where \( m \) is the mass, \( q \) the charge, and \( \kappa \) the strength of the anomalous magnetic moment term. For the electron, \( q = -e \) and \( \kappa = 0 \); for the proton, \( q = e \) and \( \kappa = 1.79 \); and for the neutron, \( q = 0 \) and \( \kappa = -1.91 \). For consistency, use the convention of Bjorken and Drell for \( F^{\mu \nu} \),

\[
F^{\mu \nu} = \frac{\partial A_{\mu}}{\partial x^{\nu}} - \frac{\partial A_{\nu}}{\partial x^{\mu}},
\]

even though this is opposite Jackson’s convention.

(a) Write out the modified Dirac Hamiltonian, and show that it is Hermitian.

(b) Show that probability is conserved, i.e.,

\[
\frac{\partial J^{\mu}}{\partial x^{\mu}} = 0,
\]

where \( J^{\mu} \) is defined exactly as for the unmodified Dirac equation, \( J^{\mu} = \bar{\psi} \gamma^{\mu} \psi \).

(c) Covariance. Suppose \( \psi(x) \) satisfies the modified Dirac equation (1), and let

\[
\psi'(x) = D(\Lambda) \psi(\Lambda^{-1} x),
\]

\[
A'^{\mu}(x) = \Lambda^{\mu}_{\nu} A^{\nu}(\Lambda^{-1} x),
\]

\[
F'^{\mu \nu}(x) = \Lambda^{\mu}_{\alpha} \Lambda^{\nu}_{\beta} F^{\alpha \beta}(\Lambda^{-1} x).
\]

Then show that \( \psi'(x) \) satisfies the modified Dirac equation (1), but with Lorentz transformed fields \( A'^{\mu}(x) \) and \( F'^{\mu \nu}(x) \) instead of the original fields.
(d) Assume $E = 0$, $B \neq 0$ (in order to see what the effective magnetic moment of the particle is). Perform a simple nonrelativistic approximation as in pp. 2–6 of the lecture notes for March 30, and show that you get the right $g$-factors for the proton and neutron.

(e) Carry out a systematic Foldy-Wouthuysen transformation for the neutron as requested by problem 4.2. Remember $q = 0$, which simplifies the calculation. Order the terms in powers of $v/c = \eta$, as done in class, and carry the expansion out to order $\eta^4$. 