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Serber	(6 pages)
Rossi	(14 pages)

Bethe had been asked to prepare a brief talk on the infrared problem for the Shelter Island conference. His presentation took place on the second day of the conference—and included a review of the papers by Bloch and Nordsieck (1937), Pauli and Fierz (1938), and Bethe and Oppenheimer (1946) (Breit 1947b). Bethe comments that after hearing Lamb's and Kramers's presentations and the discussions these generated he knew how to make a nonrelativistic calculation and was eager to do so throughout the conference.[19] He summarized these discussions in his paper:

> Schwinger and Weisskopf, and Oppenheimer have suggested that a possible explanation might be the shift of energy levels by the interaction of the electron with the radiation field. This shift comes out infinite in all existing theories, and has therefore always been ignored. However, it is possible to identify the most strongly (linearly) divergent term in the level shift with an electro-magnetic *mass* effect which must exist for a bound as well as for a free electron. This effect should properly be regarded as already included in the observed mass of the electron, and we must therefore subtract from the theoretical expression, the corresponding expression for a free electron of the same average kinetic energy. The result then diverges only logarithmically (instead of linearly) in non-relativistic theory: Accordingly, it may be expected that in the hole theory, in which the *main* term (self-energy of the electron) diverges only logarithmically, the result will be *convergent* after subtraction of the free electron expression.[6] This would set an effective upper limit of the order of mc^2 to the frequencies of light which effectively contribute to the shift of the level of a bound electron. I have not carried out the relativistic calculations, but I shall assume that such an effective relativistic limit exists. (Bethe 1947)

Bethe followed the general ideas of Kramers on mass renormalization. Explicitly this meant that in the quantum-mechanical treatment of the self-energy of a free particle one should interpret the second-order contribution ΔW, eq. (5.6.1), as a contribution to the mass of the charged particle. In other words, the zeroth and second order combine to give the energy, $\frac{p^2}{2m}$, of the particle with mass m

$$W = W_0 + \Delta W = \frac{p^2}{2m} = \frac{p^2}{2(m_0 + \mu)} \tag{5.6.2}$$

$$\Delta W = -\frac{p^2}{2m_0^2}\,\mu. \tag{5.6.3}$$

In eq. (5.6.2) $m = m_0 + \mu$ is the observed mass of the charged particle, m_0 is its "bare" mass, and

$$\mu = \frac{4}{3\pi}\frac{e^2}{\hbar c^3}\int_0^\infty dk \tag{5.6.4}$$

is the linearly divergent contribution (5.6.1). In any experiment it is only the observed mass m that can be measured, and any distinction between m_o and μ is meaningless. All observable quantities must therefore involve only the observed mass. Hence any reference to a mass other than the total observed mass must be eliminated in all equations. This is what Kramers' mass renormalization meant.

The actual calculation of the nonrelativistic Lamb shift was made on a train ride from New York to Schenectady. Bethe had stayed in New York after the Shelter Island conference to visit his mother, and had gone on to Schenectady to consult for General Electric. The calculation is straightforward (Bethe 1947). The self-energy of an electron in a quantum state m, due to its interaction with the radiation field, is

$$W = -\frac{2e^2}{3\pi\hbar c^3}\int_0^K k\,dk \sum_n \frac{|\mathbf{v}_{mn}|^2}{E_n - E_m + k}, \tag{5.6.5}$$

where $k = \hbar\omega$ is the energy of the photon that is emitted and reabsorbed by the electron, and

$$\mathbf{v} = \mathbf{p}/m = \frac{\hbar}{im}\nabla \tag{5.6.6}$$

is the velocity operator of the electron. For a free electron, $\mathbf{v}$ only has diagonal elements and (5.6.3) is replaced by

$$W_0 = -\frac{2e^2}{3\pi\hbar c^3}\int k\,dk\,\mathbf{v}^2/k. \tag{5.6.7}$$

Now W_o "represents the change of the kinetic energy of the electron for fixed momentum, due to the fact that electromagnetic mass is added to the mass of the electron. This electromagnetic mass is already contained in the experimental electron mass; the contribution (5.6.7) should therefore be disregarded. For a bound electron, $\mathbf{v}^2$ should be replaced by its expectation value $(\mathbf{v}^2)_{mm}$."[20] But since

$$\sum_n |\mathbf{v}_{mn}|^2 = (\mathbf{v}^2)_{mm}, \tag{5.6.8}$$

the relevant part of the self-energy

$$W' = W - W_0 = \frac{2e^2}{3\pi\hbar c}$$

which expression Bethe considered
This expression diverges logarithr
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Upon performing the k integration,

$$W' = \frac{2e^2}{3\pi\hbar c^3}\sum_n |$$

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$$\sum_n |p_{nm}|^2 (E_n$$

where $\psi_m(0)$ is the wave function o
shift (to this approximation) is there

$$W_n' = \frac{8}{3\pi}\left(\frac{e^2}{\hbar c}\right)^3$$

where Ry is the Rydberg energy α^2
Bethe had obtained on his arrival a
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n value $(\mathbf{v}^2)_{mm}$."[20] But since

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the relevant part of the self-energy becomes

$$W' = W - W_0 = \frac{2e^2}{3\pi\hbar c^3}\int_0^K dk \sum_n \frac{|\mathbf{v}_{mn}|^2(E_n - E_m)}{E_n - E_m + k}, \tag{5.6.9}$$

which expression Bethe considered "the true shift of the levels due to interactions." This expression diverges logarithmically as $K \to \infty$. Bethe, with characteristic self-confidence, next assumed that a relativistic hole-theoretic calculation would provide a natural cutoff for the frequency K at energies

$$K = mc^2. \tag{5.6.10}$$

Upon performing the k integration,

$$W' = \frac{2e^2}{3\pi\hbar c^3}\sum_n |\mathbf{v}_{mn}|^2(E_n - E_m)\ \ell n\ \frac{K}{|E_n - E_m|}. \tag{5.6.11}$$

Since the argument in the logarithm is very large, Bethe assumed it to be constant (independent of n) in first approximation. The sum over n can then be performed and yields

$$\sum_n |p_{nm}|^2\ (E_n - E_m) = 2\pi\hbar^2 e^2 Z\ \psi_m^2(0), \tag{5.6.12}$$

where $\psi_m(0)$ is the wave function of the electron at the position of the nucleus. The shift (to this approximation) is therefore non-zero only for S-states, and in this case

$$W_n' = \frac{8}{3\pi}\left(\frac{e^2}{\hbar c}\right)^3 \mathrm{Ry}\ \frac{Z^4}{n^3}\ \ln\ \frac{K}{< E_n - E_m >_{Ave}}, \tag{5.6.13}$$

where Ry is the Rydberg energy $\alpha^2 mc^2/2, \alpha = e^2/\hbar c$. This is the expression that Bethe had obtained on his arrival at Schenectady. He was not quite confident of its accuracy, because he was not quite sure of the correctness of a factor of $\sqrt{2}$ in his expansion of the radiation operators in terms of creation and annihilation operators. This he checked on Monday morning in Heitler's book. He also got Miss Steward and Dr. Stehn from GE to evaluate numerically $< E_n - E_m >_{Ave}$ for the $2s$ state. It was found to be 17.8 Ry, "an amazingly high value."[21] Inserting this into (5.6.13), Bethe found

$$W_{2s} \approx 1040 \text{ megacycles}, \tag{5.6.14}$$

"in excellent agreement with the observed value of 1000 megacycles" (Bethe 1947).

5.7 Relativistic Lamb Shift Calculations: 1947–1948

As soon as Bethe completed and circulated his calculation indicating that a major part of the Lamb-Retherford experimental result on the $2s$-$2p$ level shift in hydrogen could be explained as a nonrelativistic quantum-electrodynamical effect, the task at hand became to carry out a relativistic calculation using the full hole theoretic formalism to justify Bethe's introduction of the cutoff $K \approx mc^2$—a much more difficult undertaking.

French and Weisskopf at MIT continued their calculation, but now made use of Kramers' idea and thus simplified somewhat the subtraction of infinities. (Their approach will be detailed in section 5.8.) At Cornell, Bethe assigned the problem to one of his graduate students, Scalettar. Lamb at Columbia started on a hole-theoretical calculation during the early part of the summer of 1947, and was soon joined by Norman Kroll. Similarly, Fermi, who was spending the summer 1947 at Los Alamos, upon receiving a copy of Bethe's preprint explored a relativistic calculation. His first step was to understand the Bethe calculation—which he redid, in collaboration with Uehling, who was also visiting Los Alamos, but they obtained an expression for the Lamb shift which was 4/3 times larger than Bethe's formula. "The factor 4/3 [was] due . . . to the inadequacy of our assumption that the [intermediate] states can be described by plane waves,"[22] Fermi wrote Uehling after speaking to Bethe. Furthermore:

> A point that is not explained in Bethe's paper but which he explained to us in Los Alamos is the procedure for justifying that the recoil of the light quantum can be disregarded.
>
> This can actually be done by using an only slightly more complicated sum rule and I do not understand why Bethe did not follow this more complete procedure[23] in writing his paper since it would have made the result more convincing.

Fermi continued:

> The point that still is quite unsatisfactory is of course the upper limit of the logarithm in Bethe's formula (11). Apparently several people (Bethe, Weisskopf and Schwinger) have tried unsuccessfully to carry out a relativistic calculation of this upper limit. Also Teller and I tried the same and we believe that we have a method that seems to be practical though probably far from simple.
>
> This method consists in describing the [intermediate] . . . state n as plane waves plus a first approximation [Coulomb] correction which is necessary and sufficient to correct for the factor 4/3 discussed above.[24]

1. Dirac, by Feynman. (Courtesy Mi
Archives, California Institute of Technolo

2. Dirac, Su
AIP Niels Bo

Fermi gave the problem "of the electromagnetic energy level shift in the relativistic case" to Marvin L. Goldberger, who was a graduate student at Chicago at the time. Goldberger wrote Bethe in early October 1947 to ask him whether "our work is sufficiently different to warrant both Mr. Scalettar and me to work on the problem. Clearly, if our work is merely repetition of his, we will drop our program." The approach Goldberger was to employ was the Fermi-Teller proposal to use "for the intermediate state [in the hole theoretic generalization of the $\Sigma|p_{mn}|^2(E_n - E_m)$ term in the Bethe formula] the first order Coulomb perturbation of the plane waves... [since] with this device the problem appears to be not too difficult."[25]

Bethe promptly answered him and informed him of the following:

> We are using a very similar method to yours which effectively amounts to a Born approximation on the intermediate state. However, the calculation is by no means simple even with this method.... In some calculations which I did in August, I was able to... demonstrate the convergence of the result. Moreover, I found that the result is similar to the nonrelativistic case. Scalettar is now checking my arguments and especially calculating explicitly the result in order to obtain the numerical value. There are approximately twenty different terms which have to be integrated....
>
> Because of the considerable complication of the calculation I should find it desirable that the calculation be done at several places independently. You may be able to find a simple method. The main reason against further duplication is that in addition Scalettar, also Weisskopf and Lamb are engaged in similar calculations.[26]

Evidently Goldberger dropped the problem. In addition to Weisskopf and French, Kroll and Lamb, and Bethe and Scalettar, others also began work on the problem. Schwinger, who had gotten married right after Shelter Island and for nearly two months thereafter traveled throughout the United States on his honeymoon, started on such a calculation in late July; it will be presented in chapter 7. In Switzerland, Jost and Luttinger "calculated the [relativistic] line shift [for a spin 0 particle] and also found that it is finite."[27] And in Japan, Tomonaga and his collaborators, and independently Nambu, started on such a calculation early in 1948 (Tomonaga 1948; Nambu 1949; Hayakawa 1988). All these workers proved that a hole-theoretical calculation of the $2^2S_{1/2} - 2^2P_{1/2}$ displacement gave a convergent answer. However they all also concluded that the formal relativistic invariance of the Dirac, Heisenberg, Pauli, and Weisskopf formulation of quantum electrodynamics "is to some degree illusory in that all self-energies diverge logarithmically, so that the difference of two energies such as $W(2^2S_{1/2})$ and $W(2^2P_{1/2})$, although finite, is not necessarily unique" (Kroll and Lamb 1949, p. 388).

Two new problems arise in a relativistic theory. The first is to define precisely the "free electron self-energy" to be subtracted. The second is that unless care is taken, the level shift seems to come out larger than the nonrelativistic (and observed) effect by a factor of the order $1/\alpha^2$.

The most natural "free electron self-energy" expression to subtract would be $\delta mc^2\beta_{ave}$, where δmc^2 is the self-energy of an electron at rest, and β_{ave} is the expectation value of the Dirac operator β for the wave function of the bound state whose energy is to be calculated:

$$\beta_{ave} = \int \psi_0^* \, \beta\psi_0 \, d\tau. \tag{5.7.1}$$

The justification for this procedure is that in any covariant theory, the self-energy of a free electron must be equivalent to a change of its rest mass, and therefore to an extra term in the Dirac Hamiltonian of the form

$$\delta H = \beta \, \delta mc^2. \tag{5.7.2}$$

Or conversely, in the description of an electron interacting with the quantized electromagnetic field, one should write for the mass of the electron entering in the Hamiltonian

$$m_0 = m - \delta m, \tag{5.7.3}$$

where m_o is the bare mechanical mass, and m is the observed, experimental mass. The Hamiltonian is therefore

$$H = c\alpha \cdot \mathbf{p} + \beta mc^2 + H_0^{em} + H_{int} - \delta mc^2\beta, \tag{5.7.4}$$

with H_0 the Hamiltonian for the free radiation field and H_{int} the interaction Hamiltonian of the electron with the radiation field.

Now, for a free electron of energy E, the expectation value of the Dirac operator β is given by

$$u^*\beta u / u^* u = m/E. \tag{5.7.5}$$

Hence for the procedure to be satisfactory, the self-energy of the free electron must be of the form $\delta mc^2 m/E$.

Weisskopf in his 1934 paper on the electron's self-energy calculated the self-energy for an electron at rest to be equal to

$$W = \frac{3}{2\pi}\frac{e^2}{hc} mc^2 \lim_{K\to\infty} \ln\frac{2K}{mc^2} + \text{finite terms} \tag{5.7.6}$$

and indicated that "the self-ene
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$$W = \frac{e^2}{2\pi hc}\frac{mc^2}{E} m$$

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and indicated that "the self-energy of free electron in motion can be obtained by a Lorentz transformation from [this expression]. The direct calculation from the above methods is ambiguous because it leads to a difference of terms, each of which diverges quadratically. The factor of the logarithmically divergent difference of these terms depends essentially on the way in which the infinite terms are subtracted." The expression for the self-energy of a free electron of energy E which Weisskopf calculated in 1934 was in fact not relativistically covariant.

Upon starting his relativistic calculation of the level shift in hydrogen in the summer of 1947, Lamb recalculated the self-energy of a moving free electron and reported his results at a cosmic ray conference held at Brookhaven National Laboratory in September 1947.[28] He found that the divergent part of the self-energy of a free electron of energy

$$E = c\sqrt{\mathbf{p}^2 + m^2c^2} \tag{5.7.7}$$

is given by

$$\frac{3e^2}{2\pi hc}\, mc^2\, \frac{mc^2}{E} \ln \frac{K}{mc^2} + \text{finite terms},$$

where K is the upper limit at which the integral over virtual photons (including Coulomb) is cut off. However, the finite terms did not have the covariant $\frac{mc^2}{E}$ dependence. The finite part of a divergent integral depends on the exact way the integral is cut off, that is, the manner in which the upper limit is chosen. It is possible to choose the upper limit to be a suitable function of p, the electron momentum, so that the finite terms are also proportional $\frac{mc^2}{E_p}$, but such a choice would be very inconvenient for the calculation of the self-energy of a bound electron. In such calculations one "is virtually forced to take the integrals over a sphere in k-space of radius K" (Weisskopf 1939). It would be an accident if such a symmetric integration led to a covariant result for the finite terms in the self-energy. Bethe asked Lennox, then a graduate student at Cornell, to do this calculation for the free electron.[29] The result he obtained was

$$W = \frac{e^2}{2\pi hc}\, \frac{mc^2}{E}\, mc^2 \left[3 \ln \frac{2K}{2mc^2} - \frac{1}{2} + \frac{4}{9}\left(\frac{p}{mc}\right)^2\right], \tag{5.7.8}$$

the last term indicating, as expected, that the finite terms do not have the proper covariance if the integration is performed symmetrically in k space. A straightforward application of the above subtraction procedures would then lead to the (incorrect) result that there are radiative corrections to the motion of a free electron of order $\alpha(\frac{p}{mc})^2 mc^2 \sim \alpha E_{\text{kinetic}}$. For the bound state problem it would give a spurious term of order $\alpha < (\frac{p^2}{2m}) >$, that is, of order α Ry, which is $1/\alpha^2$ larger than the term Bethe calculated for the Lamb shift (Bethe 1948).

The hole-theoretic calculations of the Lamb shift were intricate and involved, and the lack of covariance made them suspect. "Most of our attempts at relativistic cut-off have run into trouble. How is the calculation of the line shift?"[30] Bethe inquired of Weisskopf in early November 1947. "Our line shift calculations have also run into some slight trouble," Weisskopf answered,"but they seem to be overcome and they change the upper limit a little compared to the value which I gave you last time. I will not publish any value at this time, because of the high fluctuations."[31]

A big breakthrough occurred with Schwinger's calculations that connected the line shift calculation with the radiative correction to the electron's magnetic moment. Rumors of Schwinger's results began circulating in early November. [32] By mid-November Weisskopf wrote Bethe: "Yes, Schwinger has a theory of corrections to the g-factor of the electron on the basis of the same idea as the line shift. It seems to work out all right and gives rise to a small positive addition to the g-value."[33]

By early December, Bethe talked about "Schwinger's new theory of Quantum Electrodynamics" at a joint theoretical seminar of Rochester and Cornell.[34] Bethe had learned of Schwinger's work from Weisskopf, who had given a seminar on it at Cornell on December 1.[35] Weisskopf in turn had been exposed to Schwinger's research at the weekly Harvard-MIT theoretical seminar at which "Julian Schwinger dominated the proceedings with his elegant and well organized lectures on his rapidly evolving 'manifestly covariant' version of quantum electrodynamics."[36] But Schwinger's insight into the connection between the anomalous moment and the Lamb shift created new problems. At the end of December 1947 Weisskopf wrote Oppenheimer: "Julian [Schwinger] and I are rather disturbed over the lack of relativistic invariance in our results. The additional magnetic moment of the electron is different when you calculate it the way Schwinger did, namely in a magnetic field, and the way we did it, namely as a split of P-states in the hydrogen atom. We did not make a mistake, since Julian has calculated both things himself, and I rely on his arithmetic more than on my own."[37] And he added in a Bohrian vein: "I am somewhat doubtful that this problem can be solved, and I think it is a limit in principle of the theory. Julian thinks that he may find a way out, but he hasn't yet."[38] Schwinger did find a way out, by formulating a fully covariant version of the theory, which will be presented in section 5 of chapter 7.

By the January 1948 meeting of the American Physical Society, much progress had been made. Feynman had developed his invariant cutoff methods to the point that the answers obtained for the anomalous magnetic moment from a calculation of radiative corrections in a Coulomb field (Lamb shift) and in an external magnetic field agreed with each other. Schwinger and Weisskopf had fully elucidated the hole-theoretic relativistic calculation of the Lamb shift and were in the process of obtaining a numerical value for it. Schwinger's method consisted "in modifying the Hamiltonian to unite the electromagnetic mass with the mechanical mass of the electron," whereas in Weisskopf's method "the elimination of the

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5.8 The French an

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$$H_{(elect)} = c\boldsymbol{\alpha} \cdot (\mathbf{p}$$

where $\mathbf{A}_0$, ϕ_0 correspond to the
itself:

$$H_{(photons}$$

$$H_{(\text{interaction})} = -e \sum_{\kappa,\lambda} B_\kappa \, \epsilon_\lambda$$

where $b_\lambda(\boldsymbol{\kappa})$, $b^*_\lambda(\boldsymbol{\kappa})$ are the usu
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method "the elimination of the

infinite self-energy" was obtained "by a suitable subtraction of the free electron mass" (Schwinger and Weisskopf 1948). This latter method was used by French and Weisskopf in their calculation of the Lamb shift, to which we now turn.

5.8 The French and Weisskopf Calculation

Spurred by Lamb's experimental finding and by Bethe's paper, French and Weisskopf intensified their efforts to calculate the $2s$-$2p$ level shift hole theoretically, a problem they had begun working on in the winter of 1946. The calculation was completed in the spring of 1948 and is contained in the thesis French submitted in May of that year to MIT. The problem addressed was described as follows: "We consider an electron in a stationary state of an externally applied time-independent electrostatic or magnetic field. The state of the vacuum in this system will be that where all the negative energy states are filled. Thus the physical situation in which we shall be interested will be that where all the negative energy states and one positive energy state are filled" (French 1948, p. 8).

The Hamiltonian for the system was taken to be

$$H = H_{electron} + H_{photon} + H_{int} + H_{coul} \tag{5.8.1}$$

$$H_{(elect)} = c\boldsymbol{\alpha} \cdot (\mathbf{p} - \frac{e_0}{c}\mathbf{A}_0) + \beta m_0 c^2 + e_0\phi_0 = H_0, \tag{5.8.2}$$

where $\mathbf{A}_0$, ϕ_0 correspond to the static external field in which the electron finds itself:

$$H_{(\text{photons})} = c\sum_{\substack{\lambda=1,2\\ \kappa}} \kappa\, b_\lambda^*(\boldsymbol{\kappa}) b_\lambda(\boldsymbol{\kappa}) \tag{5.8.3}$$

$$H_{(\text{interaction})} = -e\sum_{\kappa,\lambda} B_\kappa\, \boldsymbol{\epsilon}_\lambda \cdot \boldsymbol{\alpha}\left\{ e^{i\boldsymbol{\kappa}\cdot\mathbf{r}/\hbar}\, b_\lambda(\boldsymbol{\kappa}) + e^{-i\boldsymbol{\kappa}\cdot\mathbf{r}/\hbar}\, b_\lambda^*(\boldsymbol{\kappa})\right\}, \tag{5.8.4}$$

where $b_\lambda(\boldsymbol{\kappa})$, $b_\lambda^*(\boldsymbol{\kappa})$ are the usual annihilation and creation operators for a transverse photon of momentum $\boldsymbol{\kappa}$ and polarization λ, and B_κ

$$B_\kappa = \left[\frac{2\pi\hbar^2 c}{\kappa}\right]^{1/2} \tag{5.8.5}$$

is the normalization factor for the expansion of the radiation field operator $A(\mathbf{r})$,

$$A(\mathbf{r}) = \sum_{\substack{\lambda=1,2\\ \kappa}} B_\kappa \boldsymbol{\epsilon}_\lambda \left\{ e^{i\boldsymbol{\kappa}\cdot\mathbf{r}/\hbar}\, b_\lambda(\boldsymbol{\kappa}) + e^{i\boldsymbol{\kappa}\cdot\mathbf{r}/\hbar}\, b_\lambda^*\boldsymbol{\kappa}\right\}. \tag{5.8.6}$$

In eq. (5.8.1),

$$H_{\text{coul}} = \frac{1}{2} e^2 \int \frac{\rho(\mathbf{r})\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3r d^3r' \tag{5.8.7}$$

is the electrostatic Coulomb interaction between the electrons. The significant energy of any state was defined as the difference of the eigenvalue of H in that state and the vacuum energy. Thus French and Weisskopf calculated

$$W = W_{vacuum+1} - W_{vacuum}. \tag{5.8.8}$$

French and Weisskopf (FW) next derived an elegant formula for W valid to order e^2

$$W = W^X + W^N \tag{5.8.9}$$

$$W^X = \frac{\alpha c^2}{4\pi^2} \int \frac{d^3k}{k^2} \sum_{J\lambda}{}' \frac{A^\lambda_{0JJ0}}{(E_o - E_J - c\kappa\delta_J)} \tag{5.8.10}$$

$$W^N = -\frac{\alpha c}{2\pi^2} \int \frac{d^3\kappa}{\kappa^2} \sum_{J^-\lambda}{}' A^\lambda_{00JJ}, \tag{5.8.11}$$

where the primed sum $\sum'_\lambda$ means

$$\sum{}' F(\lambda) = F(1) + F(2) + F(3) - F(4) \tag{5.8.12}$$

and where

$$A^\lambda_{klmn} = \left\langle \Psi^*_k \, \alpha_\lambda \, e^{-i\boldsymbol{\kappa}\cdot\mathbf{r}/\hbar} \, \Psi_\ell \right\rangle \left\langle \Psi^*_m \, \alpha_\lambda \, e^{i\boldsymbol{\kappa}\cdot\mathbf{r}/\hbar} \, \Psi_n \right\rangle, \tag{5.8.13}$$

with $\alpha_1, \alpha_2, \alpha_3$ the usual Dirac matrices and $\alpha_4 = 1$. $\sum_{J^-}$ means a sum over negative energy states and $\delta_J = E_J/|E_J| = \pm 1$; k,l,m,n denote one electron states in the external field A_0, ϕ_0. FW call W^X the exchange part, and W^N the nonexchange part of W. In terms of Feynman diagrams, W^X corresponds to the contribution of the "vertex" diagrams (fig. 5.8.1) and W^N that of the vacuum polarization diagram (fig. 5.8.2).

The terms in W for $\lambda = 1, 2$ are the contributions from H_{int}, that is, from the transverse photons; those for $\lambda = 3, 4$ are those from H_{coul}.

Both W^X and W^N are divergent. The difficulty encountered by FW in their calculations before Shelter Island was that not only was W^X itself divergent but "that the difference between the values of W^X evaluated for two different states is

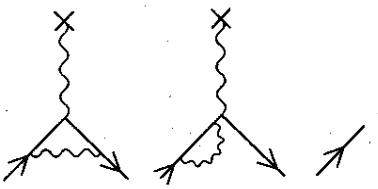

FIGURE 5.8.1

in general divergent." Thus, since t
"a new physical idea" is needed t
of two levels (French 1948, p. 27).
FW incorporated into their calcul
of the effect of the radiative corre
terms of the form

$$\delta_1 \, e_0$$

and

where δ_1 and δ_2 are of order α =
the operator O in the one-electron
tions can therefore be considered

$$H_0 = c \, \boldsymbol{\alpha} \cdot \mathbf{p} +$$

to

$$H_0' = c \, \boldsymbol{\alpha} \cdot \mathbf{p} + (1 + \delta_1)e_0$$

The effect of these additional ter
appearing in H_0 to the value

which are then identified with t
the Hamiltonian (5.8.14) can be
$m_0 + \delta m$,

$t^3 r d^3 r'$ (5.8.7)

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t formula for W valid to order e^2

(5.8.9)

$\frac{A^{\lambda}_{0JJ0}}{E_J - c\kappa\delta_J)}$ (5.8.10)

$A^{\lambda}_{00JJ},$ (5.8.11)

$(3) - F(4)$ (5.8.12)

$e^{i\kappa\cdot r/\hbar}\,\Psi_n\Big\rangle,$ (5.8.13)

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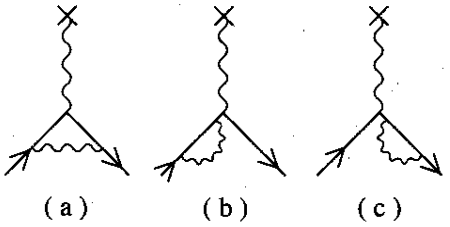

FIGURE 5.8.1

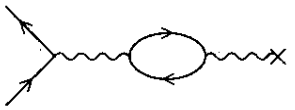

FIGURE 5.8.2

in general divergent." Thus, since the divergence depends in general upon the state, "a new physical idea" is needed to give finite values for the difference in energy of two levels (French 1948, p. 27). The new idea was provided by Kramers, which FW incorporated into their calculation in the following manner: In general, part of the effect of the radiative corrections is to generate in the perturbation energy terms of the form

$$\delta_1\, e_0 \langle [-\boldsymbol{\alpha}\cdot\mathbf{A}_0 + \phi_0] \rangle_{av}$$

and

$$\delta_2 \langle \beta m_0 c^2 \rangle_{av},$$

where δ_1 and δ_2 are of order $\alpha = \frac{e^2}{\hbar c}$, and $< O >_{av}$ means the expectation value of the operator O in the one-electron state under consideration. These radiative corrections can therefore be considered as changing the (unperturbed) Hamiltonian H_0,

$$H_0 = c\,\boldsymbol{\alpha}\cdot\mathbf{p} + \rho\, e_0[-\boldsymbol{\alpha}\cdot\mathbf{A}_0 + \phi_0] + \beta m_0 c^2, \tag{5.8.14}$$

to

$$H_0' = c\,\boldsymbol{\alpha}\cdot\mathbf{p} + (1+\delta_1)e_0[-\boldsymbol{\alpha}\cdot\mathbf{A}_0 + \phi_0] + (1+\delta_2)m_0 c^2\beta. \tag{5.8.15}$$

The effect of these additional terms is to "renormalize" the parameters e_0 and m_0 appearing in H_0 to the value

$$m = m_0(1+\delta_1) \tag{5.8.16}$$

$$e = e_0(1+\delta_2), \tag{5.8.17}$$

which are then identified with the "observable" mass and charge. Alternatively, the Hamiltonian (5.8.14) can be reexpressed in terms of the observed mass $m = m_0 + \delta m$,

$$H + c\,\boldsymbol{\alpha}\cdot\mathbf{p} + \beta mc^2 + H_{em} + H_{int} - \delta mc^2\beta, \tag{5.8.18}$$

and δm is determined so that a free electron has the correct observable mass m.

The charge renormalization term that comes from W^N had previously been discussed by Weisskopf in 1936 and can readily be evaluated (Uehling 1935).[39] After an infinite charge renormalization, the contribution of W^N, to first order in the external field, was computed by FW to be

$$\Delta W^N = -\frac{\alpha}{15\pi}\left[\frac{\hbar}{mc}\right]^2 e\left\langle\nabla^2\phi_0 - \boldsymbol{\alpha}\cdot\nabla^2\mathbf{A}_0\right\rangle_{Av}. \tag{5.8.19}$$

The mass renormalization terms were, however, more troublesome. For a free electron, that is, when $A_0 = \phi_0 = 0$, the perturbation ought to give exclusively a masslike term W_0^X, that is, a term of the form $\delta_2\beta m_0c^2$. As noted earlier, the divergent part of the self-energy,

$$\frac{3\alpha}{2\pi}\left[\beta m_0c^2\right]_{av}\int^{\infty}\frac{d\kappa}{\kappa},$$

indeed has this form. The finite terms, however, do not have this form (Weisskopf 1939)—a consequence of the divergent nature of the theory and the lack of manifest relativistic invariance in making the computations.

Two ways are open to rectify the situation: (1) devise an invariant calculational scheme that guarantees that the self-mass δm of a free electron is invariant, or (2) devise a (possibly noncovariant) method which guarantees to give zero for the self-energy of a free electron in motion calculated with the Hamiltonian (eq. 5.8.18).

French and Weisskopf chose the second method and specified that one is to subtract from the self-energy of the bound electron simply the self-energy of a free electron wave packet identical to the bound-state wave function.[40] Thus, if one expands the bound-state wave function in plane waves

$$\psi_0 = \sum_{\mathbf{p},s} a(\mathbf{p},s)\,u_s(\mathbf{p})\,e^{i\mathbf{p}\cdot\mathbf{r}/\hbar}, \tag{5.8.20}$$

where s distinguishes the states of different spin and sign of the energy ($s = 1,2,3,4$), the free electron self-energy subtracted by French and Weisskopf is given by

$$W_{free} = \sum_{\mathbf{p}}\sum_{ss'} a^*(\mathbf{p},s)a(\mathbf{p},s')(\mathbf{p}s|W|\mathbf{p}s'), \tag{5.8.21}$$

where the last factor represents the
Besides the usual diagonal element
corresponding to transitions from
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components. The method was furth
tion that a charge symmetrized[41]
the mass term to be subtracted fror

In a Lamb shift calculatio
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this difficulty, FW split the range
point $\delta\mu$. For $\kappa < \delta\mu$, the dipole
for $\lambda = 1,2$, and this term then
intermediate states can be approxi
tion evaluated. After lengthy calcu
his dissertation obtained for the c
following result for the level shift:

$$\Delta W = \frac{\alpha}{3\pi}\left(\frac{\hbar}{mc}\right)^2\left[\left\langle\nabla^2 V\right.\right.$$

$$+\frac{1}{\hbar}[$$

Here $c\kappa_0$ is the Bethe lower limit.
a spin-orbit coupling and correspo
an anomalous magnetic moment. I
spin magnetic moment $\delta\frac{e\hbar}{2mc}$, then

$$\delta\frac{e\hbar}{2mc}\boldsymbol{\sigma}\cdot(\mathcal{H} + \frac{1}{c}\mathcal{E}\times\mathbf{v}) =$$

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$s|W|\mathbf{p}s'$), (5.8.21)

where the last factor represents the matrix element of the free electron self-energy. Besides the usual diagonal elements $s = s'$, there are also "off-diagonal elements corresponding to transitions from positive to negative energy states since the expansion (5.8.20) of the Coulomb wave functions has negative energy plane wave components. The method was further refined, and FW proceeded on the assumption that a charge symmetrized[41] W_0^X evaluated for the "bound" electron state is the mass term to be subtracted from W^X (see sect. 3 of FW 1949).

In a Lamb shift calculation, the states for which the radiative corrections are calculated are nonrelativistic, that is, states whose energy, exclusive of the rest mass energy, is small compared to mc^2, and whose momenta $\bar{p}$ are small compared to $\mu = mc$. In evaluating eq. (5.8.9), Ψ_0 is treated exactly (i.e., as a state of the electron in the field A_0, ϕ_0), but the intermediate states are expanded in powers of A_0, ϕ_0, regarding A_0, ϕ_0 for these states as a perturbation on the free-particle Hamiltonian, $c\boldsymbol{\alpha} \cdot \mathbf{p} + mc^2$. This procedure is not accurate for low-lying intermediate states but is satisfactory for higher states in the continuum. To circumvent this difficulty, FW split the range of κ values at some convenient intermediate point $\delta\mu$. For $\kappa < \delta\mu$, the dipole approximation can be made in computing W^X, for $\lambda = 1, 2$, and this term then yields the Bethe contribution. For $\kappa > \delta\mu$ the intermediate states can be approximated by plane wave states, and the contribution evaluated. After lengthy calculations, done in two different ways, French in his dissertation obtained for the case of a Coulomb field, $e\phi = V$, $\mathbf{A} = 0$, the following result for the level shift:

$$\Delta W = \frac{\alpha}{3\pi}\left(\frac{\hbar}{mc}\right)^2\left[\langle\nabla^2 V\rangle_{Av}\left\{\int_{\kappa_0}^{\mu}\frac{d\kappa}{\kappa} - \ln 2 + \frac{5}{6} - \frac{1}{5}\right\} + \frac{1}{\hbar}[\nabla V \cdot \boldsymbol{\sigma} \times \mathbf{p}]_{Av}\left\{-\frac{3}{4}\right\}\right]. \tag{5.8.22}$$

Here $c\kappa_0$ is the Bethe lower limit. The second term has the characteristic form of a spin-orbit coupling and corresponds to the Schwinger result that the electron has an anomalous magnetic moment. If the electron is assumed to possess an additional spin magnetic moment $\delta\frac{e\hbar}{2mc}$, then the term

$$\delta\frac{e\hbar}{2mc}\boldsymbol{\sigma} \cdot (\mathcal{H} + \frac{1}{c}\mathcal{E} \times \mathbf{v}) = \delta\left\{\frac{e}{mc}\mathbf{s} \cdot \mathcal{H} - \frac{e\hbar}{2m^2c^2}\mathcal{E} \cdot \mathbf{r} \times \mathbf{p}\right\} \tag{5.8.23}$$

must be added to the Hamiltonian. When there is no external magnetic field present, $\mathcal{H} = 0$. Hence the second term in eq. (5.8.22) indicates that the electron behaves as if it had an additional contribution to $g/2$ of magnitude

$$\delta = \frac{\alpha}{2\pi}. \tag{5.8.24}$$

By the spring of 1948 four Lamb shift calculations had been completed: by French and Weisskopf, by Kroll and Lamb, by Schwinger, and by Feynman. The first sets of authors had used a hole-theoretic approach. Schwinger and Feynman, on the other hand, had used four-dimensional covariant methods, and had found that the radiative corrections to the motion of an electron in an external electromagnetic field $(\phi, \mathbf{A})$ due to quanta of energy greater than k_1, where

$$\mathrm{Ry} \ll k_1 \ll mc^2,$$

added to the one-particle Hamiltonian a term

$$H_1 = \frac{e^2}{3\pi\hbar c}\left(\frac{\hbar}{mc}\right)^2 \Box^2 e(\phi - \boldsymbol{\alpha}\cdot\mathbf{A})\left(\ln\frac{mc^2}{k_1} + a - \ln 2\right) - \frac{e^2}{2\pi\hbar c}\frac{e\hbar}{2mc}\beta\,(\boldsymbol{\sigma}\cdot\mathscr{H} - i\boldsymbol{\alpha}\cdot\mathscr{E}), \tag{5.8.25}$$

where $\Box^2$ is the d'Alembertian

$$\Box^2 = \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}, \tag{5.8.26}$$

$\mathscr{H}$ is the external magnetic field, $\mathscr{H} = \nabla \times \mathbf{A}$, and $\mathscr{E}$ is the external electric field. The constant a was 5/8 according to Feynman, 3/8 according to Schwinger.

If eq. (5.8.25) is combined with the effect of the quanta of energy less than k_1, the level shift for the case of a Coulomb field $(\mathbf{A} = 0, -e\phi = Ze^2/r)$ is given by

$$W = \frac{8}{3\pi}\left(\frac{e^2}{\hbar c}\right)^3 Ry\frac{Z^4}{n^3}\left(\ln\frac{mc^2}{16.721\mathrm{Ry}\,Z^2} + a - \ln 2\right)$$

for S-states (5.8.27)

$$W = \frac{8}{3\pi}\left(\frac{e^2}{\hbar c}\right)Ry\frac{Z^4}{n^3}\left(\frac{3}{8\ell(\ell+1)(2\ell+1)}\boldsymbol{\ell}\cdot\boldsymbol{\sigma} + N\right)$$

for $\ell \neq 0$ (5.8.28)

(N is the small contribution from eq. 5.6.9 for $\ell = 0$). The value of a found by these various authors is given in table 5.8.1 taken from Bethe's 1948 Solvay report.[42] The contribution $-\frac{1}{5}$ to a in the table is that due to the Uehling term.

Table 5.8.1
Results for 2s state of

Author	
Schwinger	
Feynman with polarization	5/8
Feynman without polarization	
Lamb	3/4
Weisskopf and French	5/6
Lamb without polarization	

French and Weisskopf we
then followed what Weisskopf ha
1983, p. 75). FW showed their res
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f an electron in an external elec-
reater than k_1, where

$$\left(\frac{2c^2}{k_1} + a - \ln 2\right) \tag{5.8.25}$$

(5.8.26)

1 $\mathcal{E}$ is the external electric field.
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$$\left(\frac{}{Z^2} + a - \ln 2\right)$$

for S-states (5.8.27)

$$\left(\frac{}{+1)}\ell \cdot \boldsymbol{\sigma} + N\right)$$

for $\ell \neq 0$ (5.8.28)

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ethe's 1948 Solvay report.[42] The
Uehling term.

Table 5.8.1
Results for 2s state of hydrogen, in megacycles.

Author	Value of a	Shift of $2s$	$2s=2p$
Schwinger	$3/8 = 0.375$	1003.13	1016.11
Feynman with polarization	$5/8 - 1/5 = 0.425$	1009.91	1022.89
Feynman without polarization	$5/8 = 0.625$	1037.03	1050.01
Lamb	$3/4 - 1/5 = 0.550$	1026.86	1039.84
Weisskopf and French	$5/6 - 1/5 = 0.633$	1038.15	1051.13
Lamb without polarization	$3/4 = 0.750$	1053.98	1066.96

French and Weisskopf were the first to arrive at the correct result. There then followed what Weisskopf has called a "tragicomical" episode (Weisskopf 1983, p. 75). FW showed their result to Schwinger and Feynman, who also had calculated the Lamb shift but, as indicated in table 5.8.1, had found a result differing from that of FW by a small additive numerical constant. Although in April, Feynman's and Schwinger's answer also differed from each other, by the end of the summer they had obtained the same answer. Weisskopf lost faith in the accuracy of the FW result. "The trouble was that both of them got the same result. Having both Feynman and Schwinger against us shook our confidence, and we tried to find a mistake in our calculation, without success" (Weisskopf 1983, p. 75).

In December 1948, Weisskopf wrote Oppenheimer:

> I have not too much to report about electrodynamics. The present state is as follows: Schwinger has essentially conformed his calculations to the ones of Feynman and given up any fancier requirements than the ones used in Feynman's calculation and therefore gets exactly the same result as Feynman, which as you know differs from our own. We could locate the trouble to that extent that the difference between the two methods is an integral which, with all reasonable methods of evaluating (like the Feynman method of introducing heavy light quanta) is identically zero. We do not know the rather subtle reason why we find some difference at the end. It seems to me, however, that the Schwinger-Feynman result will be the right one, since it retains relativistically invariant forms during a larger part of the calculation.[43]

Feynman, who had likewise been puzzled by the discrepancy, wrote a letter to Weisskopf shortly before Christmas 1948 indicating that he felt that the FW results were incorrect:

> Maybe this summer . . . I will try to figure out why we get a different answer for self-energy. The places where there may be trouble are:
>
> (1) to use the correction for the radiationless scattering in the self-energy problem may not be precise, (2) as you suggest, there may be some error in using a small mass for the quanta to avoid the lower limit in the integrals. I think it could all be straightened out very easily if you would calculate the radiationless scattering problem by your methods. If you still get 5/6, then I think you are wrong; but if you get 5/8 for this problem and 5/6 for the self-energy problem, I resign. In addition, if the latter turns out to be the case you will be able to see what the difference is in the two problems which leads to the different numerical value. Alternatively I could calculate this self-energy problem with much greater precision and verify that my methods of calculating the radiationless scattering first and simply transferring the results is O.K.[44]

The puzzle was finally resolved by French: "The source of discrepancy is the way in which the joining to the Bethe non-relativistic result is done." Feynman in his calculation had given the photons a small mass λ and integrated down to $k = 0$ and had obtained a level shift proportional to $(\ln m/\lambda - 3/8)\ (0|\nabla^2 V|0)$. However, care must then be exercised in calculating and joining the contributions of the longitudinal photons—for which retardation effects cannot be neglected—with the nonrelativistic result of Bethe. Feynman had not taken into account this nonrelativistic longitudinal contribution, which does not vanish when the joining is made with a nonvanishing photon mass λ.[45]

In a footnote to his positron theory paper, Feynman (1949b) noted that FW repeatedly pointed out to him that his published result for the Lamb shift (Feynman 1948b) "was in error." Feynman went on to say that: "The author feels unhappily responsible for the very considerable delay in the publication of French's result occasioned by the error." The delay in FW publishing their result resulted in Kroll and Lamb's calculation appearing in print a few months earlier than FW's[46] (Kroll and Lamb 1949).

Interestingly, the same joining mistake as Feynman's had been made by Schwinger. The delay in the publication of French and Weisskopf's result was also due to a disagreement with Schwinger's result. John Blatt who was at MIT at the time recalls asking Julian Schwinger for the reason of the disparity and he "distinctly remembers Schwinger's shrug of the shoulders, along with words somewhat

like: "Well, if you do not keep t
happen."[47] Later on, after Schw
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5.9 Radiative Cor

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like: "Well, if you do not keep the calculation explicitly covariant, anything can happen."[47] Later on, after Schwinger's mistake had been discovered Weisskopf told Blatt, "Er ist zum Kreuze gekrochen," referring to the "crawling to the cross" at Canossa in the investiture contest with the pope.[48] Schwinger also acknowledged his responsibility for the delay in the publication of FW's calculation—but much less forthrightly than Feynman had—in a letter to the editor reporting on his results on the radiative corrections for the Coulomb case (Schwinger and Feldman 1949).[49]

As Weisskopf was to remark, "Self-confidence is an important ingredient that makes for a successful physicist" (Weisskopf 1983, p. 75).

This failure of nerve robbed Weisskopf of the credit he so richly deserved in being the first, with French, to calculate relativistically the value of the Lamb shift. Weisskopf had taken seriously the discrepancy between the Dirac theory and the experimental data of Houston and R. C. Williams on the spectrum of hydrogen, and he was the first to recognize that a hole-theoretic calculation of the $2s - 2p$ level shift could be carried out. In fact, he had put French to work on the problem in October 1946, well before he knew of Lamb's experiment.

5.9 Radiative Correction to Scattering

Bethe's calculations had indicated that in hole theory, after mass renormalization, a finite value would be obtained for the radiative corrections (to order e^2) to the energy levels of an electron in a bound state in a Coulomb field.

The question immediately arose whether the radiative corrections to the cross section for the scattering of electron in a Coulomb field would also be finite. Since the cross section is an observable quantity, a satisfactory theory must give a finite value for it. That a finite result would be obtained was almost guaranteed, since a close relation exists between shifts in energy levels and radiative corrections to scattering. If $(\mathbf{p}'|V|\mathbf{p})$ is the matrix element of the unperturbed potential for scattering from $\mathbf{p}$ to $\mathbf{p}'$ and $(\mathbf{p}'|\delta V|\mathbf{p})$ the radiative corrections to it, then the level shift for the state $\psi(\mathbf{p})$ is given by

$$\Delta E = \int d^3p' \int d^3p \psi^*(\mathbf{p}')(\mathbf{p}'|\delta V|\mathbf{p})\psi(\mathbf{p}) \qquad \textbf{(5.9.1)}$$

according to perturbation theory.

Since a finite result had been obtained for ΔE it was likely that $(\mathbf{p}'|\delta V|\mathbf{p})$ would also be finite. On the other hand, Dancoff had calculated $(\mathbf{p}'|\delta V|\mathbf{p})$ in 1938/39 and had obtained a divergent expression for it (Dancoff 1939).

Oppenheimer gave the problem of calculating the radiative corrections to the radiationless scattering cross section of an electron by a Coulomb field to Hal Lewis at the end of the summer of 1947. Lewis, together with Robert Finkelstein,