Physics 221B
Spring 2008
Homework 29
Due Friday, May 16 at 5pm

Reading Assignment: Lecture notes for 5/6/08 and 5/8/08; Sakurai, *Advanced Quantum Mechanics*, pp. 146–156, 188–194. In lecture on Friday we just got started on two-photon annihilation, which is discussed by Sakurai, starting at p. 204.

I remind you that this last homework must be turned in on time (no lates allowed).

1. In this problem we treat the neutron by means of quantum field theory. The Lagrangian density for neutrons interacting with an external or c-number electromagnetic field is

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1, \tag{1}$$

where

$$\mathcal{L}_0 = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi, \tag{2}$$

and

$$\mathcal{L}_1 = -\frac{\kappa e}{4m}\bar{\psi}\,\sigma^{\mu\nu}F_{\mu\nu}\psi. \tag{3}$$

Here ψ is the Dirac quantum field describing the neutron and κ is a constant. Here we use the Jackson convention for $F_{\mu\nu}$,

$$F_{\mu\nu} = \frac{\partial A_\nu}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\nu}. \tag{4}$$

Notice the absence of a $q\not{A}$ term, since $q = 0$ for the neutron. Although the neutron is neutral, it does interact with an electromagnetic field, because of its spin and magnetic moment.

(a) Consider a localized, electrostatic potential (for example, that produced by the nucleus of an atom), given by $A^\mu = (\Phi, \mathbf{0})$, where $\Phi = \Phi(\mathbf{x})$. Calculate the differential cross section $d\sigma/d\Omega'$ for a neutron in state (ps) to scatter off the electrostatic potential into final state $(p's')$. Use unprimed variables to describe the initial state, for example, $p^\mu = (E, \mathbf{p})$, and primed variables to describe the final state, for example, $p'^\mu = (E', \mathbf{p}')$. Express the cross section in terms of the Fourier transform $\tilde{\Phi}$ of the potential Φ, using the conventions of Eq. (29.59). At this point you need not evaluate the spin matrix elements in Dirac spin space.

(b) Now suppose that the initial neutron beam is unpolarized, and we don't care about the final neutron spin state. Evaluate all spin sums and express the effective differential cross section $d\sigma/d\Omega'$ in terms of the initial momentum of the particle, the scattering angle θ, and $\tilde{\Phi}$.