Physics 221B
Spring 2008
Homework 24
Due Friday, April 4 at 5pm

Reading Assignment: Rest of Notes 37, Notes 38, lecture notes for 3/20/08; Sakurai, *Advanced Quantum Mechanics*, pp. 47–53.

1. This is a variation on Sakurai problem 2-6, p. 72. Consider the $2s \to 1s$ transition in hydrogen. The matrix elements for single photon emission are very small, so ignore them, and consider the emission of two photons. This in fact is the dominant mechanism for the $2s \to 1s$ decay. Ignore the spin of the electron (and thus the fine structure, etc), so the atomic states are $|n\ell m\rangle$. Use the dipole approximation, so $e^{i\mathbf{k}\cdot\mathbf{x}} = 1$. See Eq. (38.16) for the effective transition amplitude in second order time-dependent perturbation theory (which we used in lecture to derive the Kramers-Heisenberg formula). Remember that H_1 in that formula must be identified with $H_1 + H_2$ in our application. Let B stand for the $2s$ state, A for the $1s$ state, and I for an intermediate state. Obtain an expression for the differential transition rate,

$$\frac{dw}{d\omega\, d\Omega\, d\Omega'}, \tag{1}$$

defined by saying that

$$\frac{dw}{d\omega\, d\Omega\, d\Omega'}\Delta\omega\, \Delta\Omega\, \Delta\Omega' \tag{2}$$

is the probability per unit time for two photons to be emitted with frequencies ω, ω', with $\omega + \omega' = \omega_{BA} = (E_B - E_A)/\hbar$, with $0 \le \omega \le \omega_{BA}/2$, with ω lying in a small interval $\Delta\omega$, and with wave vectors $\mathbf{k}$, $\mathbf{k}'$ lying in two small cones of solid angles $\Delta\Omega$ and $\Delta\Omega'$. You can write your answer in the style we used for the Kramers-Heisenberg formula, with the cross section proportional to the square of a certain amplitude, which you should define. The answer is a function of ω and the directions and polarizations of the two outgoing photons. Use atomic units (it is much easier).

Indicate which intermediate states contribute to the sum. Simplify the expression to the extent that you can do the sum over polarizations and the integration over solid angle, leaving behind only a final integral over ω to get the total transition rate. This problem is relatively simple as far as two-photon processes go, since the initial and final atomic states are rotationally invariant.

2. Sakurai derives the Thomson cross section for the scattering of photons by free electrons as a limit of the Kramers-Heisenberg formula when $\hbar\omega$ is much greater than atomic energy scales (but much less than mc^2). Unfortunately, this derivation disguises the fact that the Thomson cross section is valid also for low energy photons. Make a direct derivation of the Thomson cross section, working with free electrons. Assume the photon energy satisfies $\hbar\omega \ll mc^2$ (thus you need not distinguish between the rest frame of the electron and the center of mass frame).