Physics 209
Fall 2002
Notes 2
SI and Gaussian Units

These notes are intended to help you become comfortable with the two systems of electromagnetic units that will be used in this course, SI units and Gaussian units. Jackson has a appendix in which he discusses units in complete detail, including several systems that we will not use. These notes are more discursive, and contain some hopefully useful pointers. The typical problems you will encounter regarding units are how to remember formulas in the two systems of units; how to convert formulas from one system to the other, in case you don't remember; and how to convert a numerical answer computed in one system to the other system.

We begin with a comparison of SI and Gaussian units, the two systems most commonly used by physicists. Each of these has advantages and disadvantages, which are somewhat complementary. The main advantage of SI units is popularity; SI units are used by the entire engineering world, usually by chemists, and by many physicists. For example, if you want to interface a piece of experimental apparatus with electrical equipment that is manufactured by someone else, or if you simply want to plug it into the wall, you must communicate in SI units such as Volts, Amperes, Ohms, etc., because that is the language that everyone else (including the power company) uses. On the other hand, just because most people use SI units, it does not mean that they are the best (sort of like Microsoft Windows). The main disadvantage of SI units is that they are unesthetic, unsymmetrical and inconvenient when questions of fundamental electromagnetic theory are involved. For example, SI units are awkward for expressing and understanding the theory of relativity. For such purposes Gaussian units are better, and most theoretical physicists prefer Gaussian units in general. There are no ϵ_0's and μ_0's in Gaussian units (and many with a taste for esthetics feel that the 0 subscripts just add insult to injury). Gaussian units are really the simpler system of units, the formulas of electromagnetism are simpler and easier to remember in Gaussian units, and it would be better to use them for pedagogical purposes were it not for the fact that for practical reasons students almost always have to face SI units some day.

In the battle of units, SI have gradually been winning territory against Gaussian (and other) systems, but I doubt that Gaussian units will ever be abandoned completely. Undergraduate books now seem to be mostly in SI units (they used to cover both systems), and chemists seem to have come over to the SI system (although atomic and molecular physics is certainly simpler in Gaussian units). The earlier editions of Jackson's book were in Gaussian units, but the current (third) edition has been partly converted to SI units. I think the reason was that engineers complained about the Gaussian units in earlier editions, and the publisher wanted to sell more books.

One small difference between the Gaussian and SI systems is the units used for mechanical quantities. The SI system uses MKS (meter-kilogram-second) mechanical units (energy in Joules, force in Newtons, etc.), while the Gaussian system used CGS (centimeter-gram-second) mechanical units (energy in ergs, forces in dynes, etc). The conversion between the different kinds of mechanical units involves only integer powers of 10, and is a simple matter in practice. (One Joule = 10^7 ergs, one Newton = 10^5 dynes, etc.)

So let us examine the more substantial differences between the two systems. We begin with the definition of the unit of charge. Coulomb's law in words says that the force between two charges at rest is directed along the line joining the charges, is proportional to the product of the charges with a positive force considered repulsive, and is inversely proportional to the distance between the charges. In equations we write

$$F = k_e \frac{Q_1 Q_2}{d^2}, \tag{2.1}$$

where Q_1, Q_2 are the two charges, d is the distance between them, and k_e is a constant (this is a scalar equation but gives the component of the force along the line joining the particles, with the sign convention just mentioned).

Obviously the value of the constant k_e depends on the choice of the unit of charge. One possibility is to use an arbitrary unit of charge, such as the amount of charge deposited on a standard glass rod by three rubs with a standard piece of cat fur, or the amount of charge needed to deposit so many grams of silver on an electrode in an electrochemical cell. Arbitrary systems of units like this are (or once were) common in practice. For example, the second was originally defined as $1/86,400$ of a day (the rotational period of the earth), and the original definition of a meter (by Napoleon's commission) was $1/10,000,000$ of the distance from the equator to the north pole of the earth. Later the definition of the meter was changed to the distance between two marks on a metal bar kept under careful conditions in France. With an arbitrary choice for the unit of charge, the constant k_e in Eq. (2.1) is a number that must be measured experimentally.

Another possibility is to choose the unit of charge to make the constant k_e look simple. The choice $k_e = 1$ is the simplest, and this is the choice made in the Gaussian system. Thus, in Gaussian units, the unit of charge (the *statcoulomb*) is defined as the amount of charge which produces a force of one dyne at a distance of one centimeter distance from an equal charge. As you know, in SI units the constant k_e is written $1/4\pi\epsilon_0$, which has a value that is not particularly simple. As far as electrostatics are concerned, SI units look as if the unit of charge (the *Coulomb*) was arbitrarily chosen. Actually, the Coulomb was chosen to make the magnetic force law look (somewhat) simple, a subject we will address below. Moreover, there is the question as to why (in SI units) the constant k_e (granted that its value is not simple) is written in such a complicated way. As we will explain below, a rationale can be given for the 4π, at least.

As far as electrostatics are concerned, the $1/4\pi\epsilon_0$ that occurs everywhere in SI formulas is just a way of writing k_e, which has the value 1 in Gaussian units. Therefore to convert a formula of electrostatics from SI to Gaussian units you just set $4\pi\epsilon_0 = 1$. However, there are further complications when magnetic fields are involved, and even in electrostatics the rule is not so simple if **D** is involved (see below).

In the Gaussian system, the unit of charge (the statcoulomb) is not independent of mechanical units, as we see by doing a dimensional analysis on the Gaussian form of Coulomb's law,

$$F = \frac{Q_1 Q_2}{d^2}. \tag{2.2}$$

This gives the dimensional relationship,

$$Q = \left(\frac{ML^3}{T^2}\right)^{1/2}, \tag{2.3}$$

where Q means charge, M means mass, L means distance, and T means time. Because of the fractional powers that occur, we may wish to treat charge as an independent unit, but we can get rid of it if we want and express everything (magnetic fields, resistance, etc) purely in mechanical units. For example, in Gaussian units, a resistance is measured in sec/cm^2, whereas in SI units it is measured in kg-meter/sec-Coul^2 (that is, Ohms). The SI Ohm is an example of a principle that sometimes works in converting between the two systems; if you take its dimensions, namely ML/TQ^2, and eliminate Q^2 using Eq. (2.3), you get T/L^2, the dimensions of the Gaussian unit of resistance. However, this does not work for other quantities (notably magnetic fields), for reasons explained momentarily.

To repeat, we can get rid of charge as an independent unit by choosing the unit of charge so that $k_e = 1$. Similarly, if we choose units of distance so that $c = 1$, then we can get rid of distance as an independent unit, which can be expressed as time. No need to stop there. Particle physicists like "natural" units, in which $\hbar = c = 1$, and in which mass has dimensions of inverse length (or time). As for electric charge, it is dimensionless in natural units, as we see by setting $L = T = 1/M$ in Eq. (2.3), and in fact the charge on the proton (in natural units) is $e = \sqrt{\alpha} = 1/\sqrt{137}$, where α is the fine structure constant. There are also Planck units, in which $G = \hbar = c = 1$ (G is Newton's constant of gravitation), in which all measurements of mass, distance, time, energy, momentum, charge, etc., are made in terms of dimensionless numbers. For example, one centimeter is 2.47×10^{32} in Planck units. Planck units would simplify the fundamental laws of physics more than any other system (they would get rid of the most constants), but most people would not like the large numbers that result for ordinary measurements.

Once the unit of charge is defined, then charge density ρ is defined as charge per unit volume, and current density $\mathbf{J}$ as charge per unit area per unit time. These are the same definitions in both systems of units (although ρ comes out in Coulombs/meter^3 in SI units and statcoulombs/cm^3 in Gaussian units, etc). Thus, the continuity equation,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0, \tag{2.4}$$

is the same in both systems of units.

We now consider the definition of the electric field. Let us begin with an imaginary experiment, in which a set of charges are present in some region of space. We treat these charges as if they were the only charges in the universe (any others are assumed to be far enough away that they have no effect on our experiment). The charges may be in an arbitrary state of motion. We pick out one charge Q, call it the "test" charge, and measure the force on it under different circumstances. When Q is stationary, we call the force $\mathbf{F}_e$ on Q the "electric" force, and we find experimentally that it is proportional to Q. It also depends on the position $\mathbf{x}$ of Q and the time t at which the measurement is made. Then we define the electric field $\mathbf{E}(\mathbf{x}, t)$ as the force per unit charge on Q, so that

$$\mathbf{F}_e = Q\mathbf{E}(\mathbf{x}, t). \tag{2.5}$$

This definition of electric field (electric force per unit charge) is the same in both SI and Gaussian units.

Next, in the case of static electric fields, we know that the energy required to move a test charge to some position in space, relative to a reference or ground, is independent of the path followed. This means that

$$\nabla \times \mathbf{E} = 0, \tag{2.6}$$

an equation of electrostatics that is the same in both SI and Gaussian units. The energy in question is of course the potential energy, which is also proportional to the test charge, so we define the potential energy per unit charge Φ as the quantity such that

$$\mathbf{E} = -\nabla \Phi, \tag{2.7}$$

another equation of electrostatics that is the same in SI and Gaussian units. (Somewhat confusingly, Φ is often called the "potential," even though it is really a potential energy per unit charge.) In SI units, potential is measured in Joules/Coulomb or *Volts*, whereas in Gaussian units it is ergs/statcoulomb or *statvolts*.

A common problem in practice is that you do a numerical calculation in Gaussian units, and you need to convert the answer to SI units. For this purpose, it helps to remember that one statvolt is 300 Volts (where "300" really means 299.79..., see Jackson's appendix). This often happens because physicists like to measure energies in electron-Volts, really an SI (or hybrid) unit.

To go back to the imaginary experiment above, suppose now that the test charge Q is in motion with velocity $\mathbf{v}$. Then we define the "magnetic force" on Q as the total force $\mathbf{F}$ minus the electric force $\mathbf{F}_e$ (the force that Q would experience if it were stationary),

$$\mathbf{F}_m = \mathbf{F} - \mathbf{F}_e. \tag{2.8}$$

Then we can take the following as experimental facts. First, $\mathbf{F}_m$ (like $\mathbf{F}_e$) is proportional to Q. Second, it is linear in the velocity $\mathbf{v}$, that is, if we do several measurements with the same Q but with different velocities, we find that if velocity $\mathbf{v}_1$ gives magnetic force $\mathbf{F}_{m1}$ and velocity $\mathbf{v}_2$ gives magnetic force $\mathbf{F}_{m2}$, then velocity $a_1\mathbf{v}_1 + a_2\mathbf{v}_2$ gives magnetic force $a_1\mathbf{F}_{m1} + a_2\mathbf{F}_{m2}$. Third, the magnetic force is orthogonal to the velocity, $\mathbf{v}\cdot\mathbf{F}_m = 0$. The first and second properties mean that there is a matrix M, a function of the position $\mathbf{x}$ of Q and the time t of the measurement, such that

$$\mathbf{F}_m = Q\mathsf{M}\mathbf{v}, \tag{2.9}$$

while the third property means that M is antisymmetric, $M_{ij} = -M_{ji}$. An antisymmetric matrix has 3 independent components, which are conveniently expressed in terms of a vector $\mathbf{M} = (M_x, M_y, M_z)$ by

$$\mathsf{M} = \begin{pmatrix} 0 & M_z & -M_y \\ -M_z & 0 & M_x \\ M_y & -M_x & 0 \end{pmatrix}, \tag{2.10}$$

so that Eq. (2.9) has the form

$$\mathbf{F}_m = Q\mathbf{v}\times\mathbf{M}. \tag{2.11}$$

Now we define the magnetic field. Unlike the definition of the electric field, the magnetic field is defined differently in the SI and Gaussian systems. In the SI system, the magnetic field is defined by $\mathbf{B} = \mathbf{M}$, whereas in the Gaussian system, it is defined by $\mathbf{B} = c\mathbf{M}$. Thus, the magnetic fields in the two systems do not have the same dimensions, even after the relation (2.3) is accounted for (which might have been used to get rid of charge as an independent unit in the Gaussian system). This is really the biggest difference between the two systems of units, and the one that causes the most confusion. In fact, the dimensions of magnetic field in Gaussian units are velocity times the dimensions of magnetic field in SI units. Thus, the force law is different in the two systems, it is

$$\mathbf{F} = Q(\mathbf{E} + \mathbf{v}\times\mathbf{B}) \tag{2.12}$$

in SI units, and

$$\mathbf{F} = Q(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{B}) \tag{2.13}$$

in Gaussian units.

The SI force law looks simpler, but the Gaussian force law illustrates an important fact, namely, that $\mathbf{E}$ and $\mathbf{B}$ have the same dimensions in Gaussian units. This is one of the main advantages of Gaussian units, especially when relativity or other fundamental physics is considered, since there are many fundamental symmetries connecting $\mathbf{E}$ and $\mathbf{B}$ that are more symmetrical and elegant (and easy to remember) in Gaussian units. Actually, as we will see below, all the fields, $\mathbf{E}$, $\mathbf{B}$, $\mathbf{D}$, $\mathbf{H}$, $\mathbf{P}$ and $\mathbf{M}$ have the same dimensions in Gaussian units, but they have different dimensions in SI units. Thus, in Gaussian units, any time you have some kind of conversion between two fields, the conversion factors are dimensionless (for example, $\mathbf{E}$ is converted to $\mathbf{B}$ by relativity, $\mathbf{D}$ is proportional to $\mathbf{E}$ in a linear dielectric, etc).

The SI unit of magnetic field is the *Tesla*, while the Gaussian unit is the *Gauss*. The conversion between them is one Tesla = 10^4 Gauss, a conversion factor that is so easy to remember that many people who normally speak SI units will refer to magnetic fields in Gauss. Notice that since $\mathbf{E}$ and $\mathbf{B}$ have the same dimensions in Gaussian units, a Gauss is really the same as a statvolt/cm (the Gaussian unit of electric field).

The Maxwell equation $\nabla \cdot \mathbf{B} = 0$ is homogeneous in $\mathbf{B}$ and so is the same in both systems of units. This implies that there exists a vector potential $\mathbf{A}$ such that $\mathbf{B} = \nabla\times\mathbf{A}$, and this is the definition of $\mathbf{A}$ in both the Gaussian and SI systems. However, since $\mathbf{B}$ has different dimensions in the two systems, so does $\mathbf{A}$. This is an example of how the different definitions for $\mathbf{B}$ percolate through the whole system and causes different definitions for all the other magnetic quantities, $\mathbf{A}$, $\mathbf{m}$, $\mathbf{M}$, $\mathbf{H}$, etc.

For example, $\mathbf{A}$ expressed in terms of the source current in Coulomb gauge in magnetostatics is

$$\mathbf{A}(\mathbf{x}) = \frac{1}{4\pi\epsilon_0}\frac{1}{c^2}\int d\mathbf{x}'\,\frac{\mathbf{J}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} \tag{2.14}$$

in SI units, and

$$\mathbf{A}(\mathbf{x}) = \frac{1}{c}\int d\mathbf{x}'\,\frac{\mathbf{J}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} \tag{2.15}$$

in Gaussian units. The constant prefactor on the integral in the SI equation (2.14) would usually be written $\mu_0/4\pi$, but it has been written in terms of ϵ_0 here so that you can see that even with $4\pi\epsilon_0 = 1$, the SI and Gaussian formulas differ by a factor of $1/c$. The different definitions of $\mathbf{A}$ in the two systems makes the expression for $\mathbf{E}$ in terms of the potentials different in the two systems. In SI units the expression is

$$\mathbf{E} = -\nabla\Phi - \frac{\partial \mathbf{A}}{\partial t}, \tag{2.16}$$

whereas in the Gaussian system it is

$$\mathbf{E} = -\nabla\Phi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}. \tag{2.17}$$

Notice that Φ and $\mathbf{A}$ have the same dimensions (statvolts) in the Gaussian system; if you remember this, you can see where to put the $1/c$ in Eq. (2.17). Generally the Gaussian system does a better job of balancing dimensions than does the SI system.

In the imaginary experiments above we did not say how the force on the test charge Q is specified by the rest of the charges (the "source" charges). Two cases are simple. If the source charges are stationary (and have been so for a long time), then the electric force on the test charge is given by

$$\mathbf{F}_e = k_e Q \int d\mathbf{x}'\,\rho(\mathbf{x}')\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3}, \tag{2.18}$$

where ρ is the source charge density, k_e is the electric constant introduced in Eq. (2.1) and $\mathbf{x}$ is the position of the test charge. If the sources charges are in motion but the source current is steady (and has been so for a long time), then the magnetic force on the test charge is given by

$$\mathbf{F}_m = k_m Q\mathbf{v}\times \int d\mathbf{x}'\,\mathbf{J}(\mathbf{x}')\times\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3}, \tag{2.19}$$

where $\mathbf{J}$ is the source current density and k_m is a new constant (the magnetic constant). Equations (2.18) and (2.21) have been written purely in terms of forces, without reference to the electric or magnetic fields (the latter of which is defined differently in the two systems of units), because we wanted to have expressions for the electric and magnetic forces that were as independent of conventions as possible. No assumptions have been made about the unit of charge or the definitions of the electric and magnetic fields in these equations.

Taking the ratio, dimensionally speaking, of Eqs. (2.18) and (2.19) shows that the ratio k_e/k_m has dimensions of velocity^2. Therefore the value of this ratio is independent of the choice made for the unit of charge. In fact, both k_e and k_m can be measured in laboratory experiments (using some arbitrary unit of charge, if necessary), and their ratio computed. This was done in the early history of electromagnetism, and it was found that

$$\frac{k_e}{k_m} = c^2, \tag{2.20}$$

to experimental accuracy. Now we believe that the relation (2.22) is exactly true (at least, it is a fundamental tenet of electromagnetic theory). Thus, the electric and magnetic force constants are not independent. In Gaussian units, Eq. (2.20) implies $k_m = 1/c^2$, and in SI units, where k_m is usually written $\mu_0/4\pi$, it implies $\epsilon_0\mu_0 = 1/c^2$. The situation is summarized in the table. Notice that in SI units, the three quantities ϵ_0, μ_0 and c are not independent. One could get rid of one of them, but not in a symmetrical manner, so in practice all three are retained (meaning that formulas can be written in nonunique ways). In Gaussian units, there is only c.

	k_e	k_m
Gaussian	1	$\frac{1}{c^2}$
SI	$\frac{1}{4\pi\epsilon_0}$	$\frac{\mu_0}{4\pi}$

Table 2.1. Electric and magnetic force constants in the different systems of units.

In SI units, the unit of charge (the Coulomb) is chosen so as to make the magnetic force constant $k_m = \mu_0/4\pi$ take on the value $4\pi \times 10^{-7}$ (exactly). One could say this is a way of making the magnetic force law look "simple" (instead of the electric force law, the choice made in Gaussian units). Of course, $4\pi \times 10^{-7}$ is not as simple as 1. The 4π can be explained in a reasonable manner (see below), but I do not know the reason for the 10^{-7}. I do not know the history of SI units, but I suspect that it was decided to choose the unit of charge to simplify the magnetic force law because it is experimentally easier to set up accurate standards with magnetic forces than with electric forces. And I suspect that the 10^{-7} was introduced to make the Ampere (one Coulomb/sec) a reasonable unit for simple laboratory experiments and common electrical devices.

Here it is worthwhile to summarize the microscopic Maxwell equations in the two systems of units, before discussing the auxiliary (macroscopic) fields **P**, **M**, **D** and **H**. The

Maxwell equations in SI units are

$$\nabla \cdot \mathbf{E} = \rho/\epsilon_0, \tag{2.21a}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{2.21b}$$

$$\nabla\times\mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \tag{2.21c}$$

$$\nabla\times\mathbf{B} - \epsilon_0\mu_0\frac{\partial \mathbf{E}}{\partial t} = \mu_0\mathbf{J}, \tag{2.21d}$$

while in Gaussian units they are

$$\nabla \cdot \mathbf{E} = 4\pi\rho, \tag{2.22a}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{2.22b}$$

$$\nabla\times\mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = 0, \tag{2.22c}$$

$$\nabla\times\mathbf{B} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = \frac{4\pi}{c}\mathbf{J}. \tag{2.22d}$$

The Maxwell equations are more symmetrical in Gaussian units (notice particulary Faraday's and Ampere's laws, the two curl equations). If you forget where the $1/c$ factors go in the Gaussian Maxwell equations, it is easy to figure them out once you remember that **E** and **B** have the same dimensions.

As for the 4π's, it is a fact that in an electromagnetic system of units you can put the 4π's into the force laws (Coulomb's and Biot-Savart's) or you can put them into the Maxwell equations. If you suppress them one place, they will pop up in the other. The SI system has chosen to put the 4π's into the force laws, and the Gaussian system has chosen to put them into Maxwell equations. If you don't like the 4π's in the Gaussian Maxwell equations (and you are willing to have them in the force laws), then you can get what you want with a simple modification of the Gaussian system which preserves the symmetry between **E** and **B** and does not bring back the unattractive ϵ_0's and μ_0's. This system is called the Heaviside-Lorentz system, which particle theorists prefer over the Gaussian system because it gets rid of the 4π's in the field Lagrangian. We will not use Heaviside-Lorentz units in this course, but they are discussed in Jackson's appendix.

Now we come to electric and magnetic moments. Two useful facts to remember are the following. First, the definition of the electric dipole moment **p** is the same in both the SI and Gaussian systems (it is the charge-weighted position vector), so that **p** has dimensions of charge times distance (QL) in both systems. Second, the expression for the energy of an electric or magnetic moment in an external field is given by

$$W = -\mathbf{p}\cdot\mathbf{E} \qquad \text{or} \qquad W = -\mathbf{m}\cdot\mathbf{B} \tag{2.23}$$

in *both* SI and Gaussian units. Since **E** has the same dimensions in both systems of units, so does **p** (as just noted). As for a magnetic moment **m**, it has the dimensions of energy per unit **B** in both systems, but since **B** has different dimensions, the dimensions of magnetic moment otherwise expressed will be different (there is a factor of c difference between the two). This is the explanation for the factor of c difference in the two formulas for the Bohr magneton in the two systems,

$$\mu_B = \frac{e\hbar}{2mc} \quad \text{(Gaussian), or} \quad \frac{e\hbar}{2m} \quad \text{(SI).} \tag{2.24}$$

In Gaussian units, since **E** and **B** have the same dimensions, it follows from Eq. (2.23) that **p** and **m** have the same dimensions (namely, charge times distance or QL). Notice that in Gaussian units, the Bohr magneton has dimensions of charge (e) times distance ($\hbar/mc$, the Compton wavelength of the electron). In SI units **p** and **m** have different dimensions.

Next, it is useful to remember that the dipole moment densities **P** and **M** are defined as the electric or magnetic dipole moment per unit volume in *both* systems of units, although since the magnetic dipole moment has different dimensions in the two systems, so also does the magnetic dipole moment density **M** (also known as the magnetization). In Gaussian units, since **p** and **m** have the same dimensions, so do **P** and **M**. In fact (as noted above), these also have the same dimensions as **E** and **B**.

You will have noticed that in the electric or electrostatic side of electromagnetic theory, the two systems agree mostly, with the understanding that $4\pi\epsilon_0$ is replaced by unity in the Gaussian system. This includes the definitions of **E**, Φ, **p**, and **P**. It also includes the bound charge density in polarized dielectrics,

$$\rho_b = -\nabla \cdot \mathbf{P}, \tag{2.25}$$

which is the same in both systems. However, on the magnetic side, definitions generally differ by a factor of c or $1/c$, including **B**, **A**, **m**, and **M**. This difference extends to the bound current in magnetic materials, which in SI units is

$$\mathbf{J}_b = \nabla\times\mathbf{M} + \frac{\partial \mathbf{P}}{\partial t}, \tag{2.26}$$

and in Gaussian units is

$$\mathbf{J}_b = c\nabla\times\mathbf{M} + \frac{\partial \mathbf{P}}{\partial t}. \tag{2.27}$$

The auxiliary fields **D** and **H** are obtained by expressing Maxwell's equations in terms of the free charges and currents only (using Eqs. (2.25)–(2.27) to eliminate the bound charges and currents). The definitions are different in the two systems. In the SI system we define

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}, \tag{2.28}$$

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M}, \tag{2.29}$$

so that Maxwell's equations become

$$\nabla \cdot \mathbf{D} = \rho_f, \tag{2.30a}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{2.30b}$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0, \tag{2.30c}$$

$$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}_f. \tag{2.30d}$$

In the Gaussian system we define

$$\mathbf{D} = \mathbf{E} + 4\pi \mathbf{P}, \tag{2.31}$$

$$\mathbf{H} = \mathbf{B} - 4\pi \mathbf{M}, \tag{2.32}$$

which cause Maxwell's equations to become

$$\nabla \cdot \mathbf{D} = 4\pi \rho_f, \tag{2.33a}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{2.33b}$$

$$\nabla \times \mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = 0, \tag{2.33c}$$

$$\nabla \times \mathbf{H} - \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} = \frac{4\pi}{c}\mathbf{J}_f. \tag{2.33d}$$

Unfortunately, the definition of **D** in the two systems does not obey the rule that Gaussian equations in electrostatics are obtained from SI equations by setting $4\pi\epsilon_0 = 1$. Note that in the Gaussian system, **E**, **B**, **P**, **M**, **D** and **H** all have the same dimensions.