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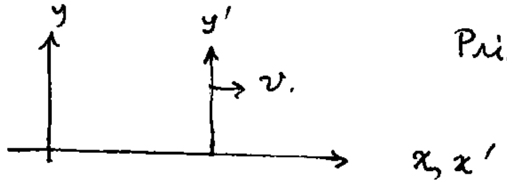
Now special relativity.

Background, inconsistency in fundamental principles underlying Newtonian mech. and EM theory.

Newt. mech. : Covariant under Galilean transfs.



Primed, unprimed frames.

$$\left.\begin{aligned} x' &= x - vt \\ t' &= t \end{aligned}\right\} \text{ Galilean transformation.}$$

$\vec{F} = m\vec{a}$ retains same form in both frames. (Covariant.)

So no meaning in Newt. mech. to absolute posn or velocity, all frames equivalent. A. Gal. transf. can also include rotations.

But Maxwell eqns not covariant under Gal. transfs. (Hence privileged frame, hence "ether".)

Discovered by Poincaré, $\underbrace{\text{form-invariant}}_{\text{Maxwell eqns}}$ under Lorentz transformations.

But Poincaré and Lorentz apparently didn't understand the transformation of time. Einstein was first to understand this. He derived Lorentz transf. from his postulates, based on physical insight.

Einstein's postulates:

1. All inertial frames equivalent.
2. Constancy of speed of light.

Now to derive Lorentz transformation from Einstein's postulates.

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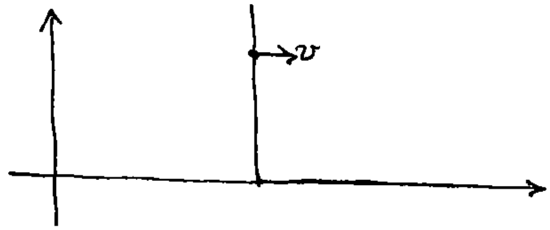


assume origins coincide, $x=x'=0$, at $t=t'=0$.

<u>assume</u>
$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \begin{pmatrix} A & B \\ D & E \end{pmatrix}\begin{pmatrix} x \\ t \end{pmatrix}$$
linear transformation req'd to map free particle orbits into free particle orbits (straight lines w. const. velocity).

<u>Use facts</u>:

$$\left.\begin{array}{lll} ① & x'=0 \Rightarrow & x=vt \\ ② & x=0 \Rightarrow & x'=-vt' \\ ③ & x=ct \Rightarrow & x'=ct' \end{array}\right\} \text{do algebra, } \Rightarrow \begin{pmatrix} A & B \\ D & E \end{pmatrix} = \gamma \begin{pmatrix} 1 & -v \\ -v/c^2 & 1 \end{pmatrix}$$

where $\gamma = A$ is still undetermined.

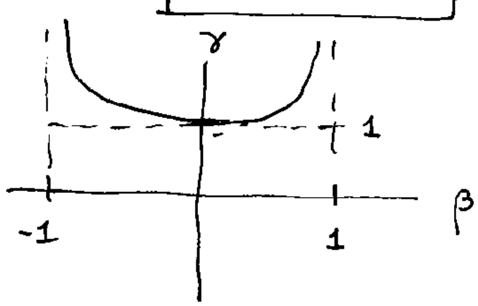
Presumably $\gamma = \gamma(v)$. Invert transformation,

$$\begin{pmatrix} x \\ t \end{pmatrix} = \underbrace{\frac{1}{\gamma(v)(1-v^2/c^2)}}_{\text{must be } \gamma(-v)} \begin{pmatrix} 1 & v \\ v/c^2 & 1 \end{pmatrix}\begin{pmatrix} x' \\ t' \end{pmatrix}.$$

But must have $\gamma(v) = \gamma(-v)$, otherwise could tell which direction. (space isotropic).

$$\Rightarrow \boxed{\gamma = \frac{1}{\sqrt{1-v^2/c^2}}}$$
usual defn. of factor γ in relativity.

Also define $\beta = \frac{v}{c}$, $\gamma = \frac{1}{\sqrt{1-\beta^2}}$.



$\gamma \geq 1$. So you get

$$\boxed{\begin{aligned} x' &= \gamma(x - vt) \\ t' &= \gamma\left(t - \frac{v}{c^2}x\right) \end{aligned}}$$
1D Lorentz transf.

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Similar argument for y coordinate. If $y' = At + Bx + Cy$,

Require: $y = 0 \Rightarrow y' = 0 \Rightarrow A = B = 0.$

$$y' = Cy, \qquad C = C(v) \text{ presumably.}$$

$$y = C(-v)y', \qquad C(v)C(-v) = 1. \qquad \text{But } C(-v) = C(v), \text{ equiv. of frames.}$$

$$C(v)^2 = 1, \quad C(v) = \pm 1. \quad \text{But } C(0) = 1.$$

Hence $y' = y$, and sim. $z' = z$.

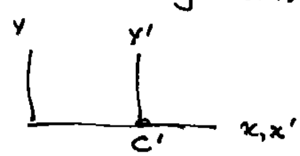
Summarize Lorentz transformation in x-direc:

$$\left.\begin{aligned} t' &= \gamma\left(t - x\frac{v^2}{c^2}\right) \\ x' &= \gamma(x - vt) \\ y' &= y \\ z' &= z \end{aligned}\right\}$$

Now draw some of the most important conclusions from this.

1. Time Dilation

Let unprimed frame be filled with synchronized clocks.
Let origin of primed frame contain a clock C' (moving clock).
When origins cross, $t = t' = 0$.
When O' has coordinate $x = vt$,

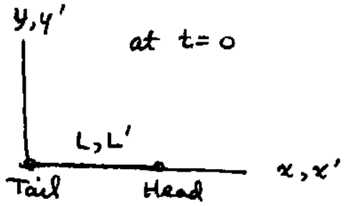


$$t' = \gamma t\left(1 - v^2/c^2\right) = \frac{t}{\gamma}.$$

$t' < t$, moving clock runs more slowly.

Paradox: doesn't $t' = \frac{t}{\gamma}$ distinguish coord. systems? Why not $t = \frac{t'}{\gamma}$?
(Later).

(2) <u>Length</u> <u>Contraction</u> A rod of length L' is fixed in moving frame. Has len. L in stat. frame. Explain meaning of length of moving object.



Moving Frame, Head: $x' = L'$, any t'

Stat. Frame, Head: $x = L$ at $t = 0$.

Plug in

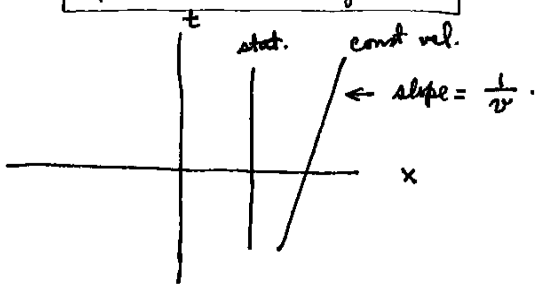
$x' = \gamma(x - vt) \Rightarrow L' = \gamma L$

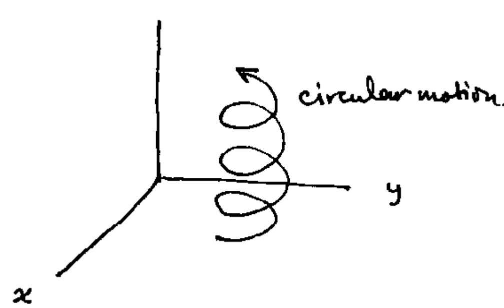
$L = \frac{L'}{\gamma}$, $L < L'$.

Moving rod is contracted in direction of motion by factor γ.

Another app. paradox: Apparent asymmetry $L < L'$ seems to say frames not equiv.

$\boxed{\text{Space-time diagrams.}}$





1+1

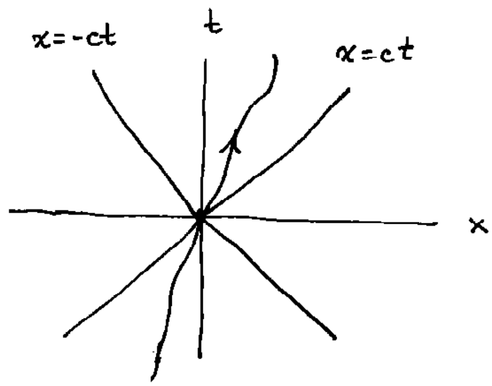
2+1

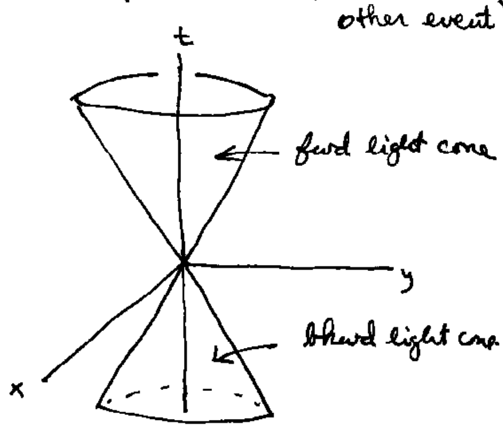
Can't draw 3+1.

<u>Event</u>: point of space-time.

<u>World line</u>: History of point or particle.

<u>Line Cone</u>: Locus of light pulse emitted at $x = y = z = t = 0$. (or at some other event)





particle with $v < c$ remains always inside light cone.

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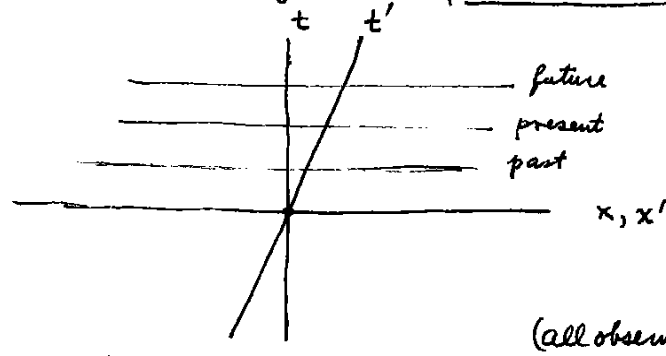
You can use space-time diagrams in <u>pre-relativistic physics</u>, if you want.

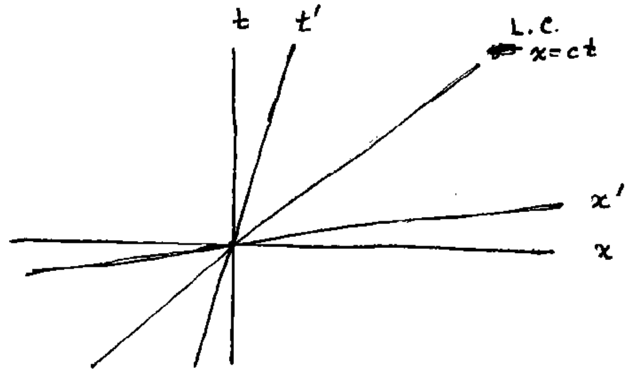


$$\left.\begin{aligned} x' &= x - vt \\ t' &= t \end{aligned}\right\} \text{ Galilean}$$

Const. time slices are horizontal lines. (Set of simultaneous events)

(all observers) Everyone agrees on which events are simultaneous.

But in relativity, simultaneity depends on observer.



$$\left.\begin{aligned} x' &= \gamma(x - vt) \\ t' &= \gamma\left(t - \tfrac{v}{c^2}x\right) \end{aligned}\right\} \text{ Lorentz}$$

x-axis = (events $t=0$) simultaneous in unprimed frame

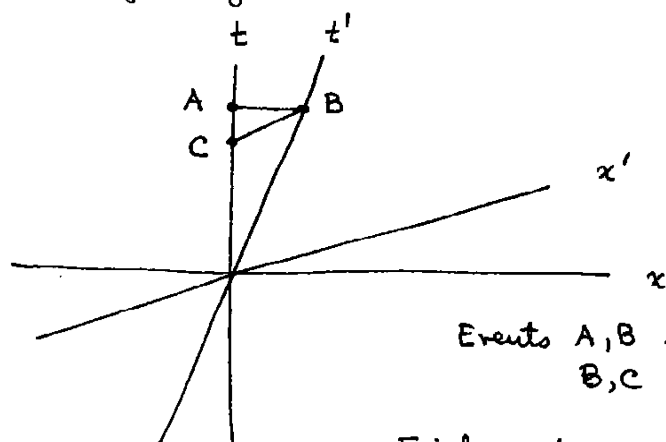
x′-axis = (events $t'=0$) " " primed frame.

Relativity of simultaneity explains various "paradoxes".

Eg. Apparent asymmetry in time-dilation.

t-axis = world line of clock C

t′-axis = " " " " C′



Events A, B simult. in unprimed frame

B, C " " primed frame.

Find $t_A = \gamma t'_B$

$t'_B = \gamma t_C$

Can generalize Lorentz transformation for velocity in arbitrary direction, $\vec{v} = v\hat{v}$, not just $\vec{v} = v\hat{x}$. Just change notation,

$$x \longrightarrow x_{\parallel} = \hat{v}\cdot\vec{x} = \frac{\vec{v}\cdot\vec{x}}{v}$$

$$\begin{pmatrix} y \\ z \end{pmatrix} \rightarrow \vec{x}_{\perp} = \vec{x} - \frac{\vec{v}(\vec{v}\cdot\vec{x})}{v^2}.$$

Then:

$$t' = \gamma\left(t - \frac{v}{c^2}x_{\parallel}\right) = \gamma\left(t - \frac{\vec{v}\cdot\vec{x}}{c^2}\right).$$

$$x'_{\parallel} = \gamma(x_{\parallel} - vt), \quad \vec{x}'_{\parallel} = \gamma(\vec{x}_{\parallel} - \vec{v}t)$$

$$\vec{x}'_{\perp} = \vec{x}_{\perp}.$$

Add these up, rearrange, you find

$$\boxed{\begin{aligned} t' &= \gamma\left(t - \frac{\vec{v}\cdot\vec{x}}{c^2}\right) \\ \vec{x}' &= \vec{x} + (\gamma-1)\frac{\vec{v}\cdot\vec{x}}{v^2}\vec{v} - \gamma\vec{v}t \end{aligned}}$$

Lorentz transf. in arbitrary direction.

$\vec{v}$ = velocity of primed frame wrt unprimed frame.