Now Clausius-Mossotti, illustrates some issues involved in microscopic model.

For single molecule, $\vec{p} = \varepsilon_0 \gamma \vec{E}$ like H_2, He, O_2 not HCl etc., H_2O

γ = molecular polarizability.
Can calculate γ from Stark effect.

So, if N = # molecules/vol., you would expect (naively),

$$\vec{P} = N\varepsilon_0 \gamma \vec{E}, \quad \text{but since } \vec{P} = \varepsilon_0 \chi_e \vec{E}, \quad \chi_e = N\gamma.$$

This is true for dilute gasses but not otherwise ...

This $\vec{E}$ is microscopic $\vec{E}$ (seen by individ. molecule)

This $\vec{E}$ is macroscopic avg. (defn. of χ_e).

So distinguish:

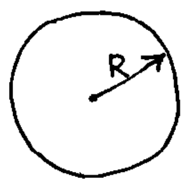
$\vec{e}$ = exact microscopic field, = Σ fields due to pt. charges.

$\vec{E}$ = macroscopic avg = field in continuum model for $\vec{P}$

$$= -\nabla\Phi, \quad \Phi = \frac{1}{4\pi\varepsilon_0}\int dv' \, \vec{P}(\vec{x}') \cdot \left(\frac{\vec{x}-\vec{x}'}{|\vec{x}-\vec{x}'|^3}\right).$$

Now take given molecule at center of a (mesoscopic) sphere, of radius R.

Break both $\vec{e}$ and $\vec{E}$ into fields due to near $(r<R)$ and far $(r>R)$ charges:



$$\left.\begin{aligned} \vec{e} &= \vec{e}_{near} + \vec{e}_{far} \\ \vec{E} &= \vec{E}_{near} + \vec{E}_{far}. \end{aligned}\right\}$$

Now,

$\vec{e}_{far} = \vec{E}_{far}$ (continuum model ok if you are far away).

$\vec{E}_{near} = -\dfrac{\vec{P}}{3\varepsilon_0}$ (result for uniformly polarized sphere)

Now,

$$\vec{p} = \gamma \varepsilon_0 \vec{e}$$

$$\begin{aligned}\vec{e} &= \vec{e}_{near} + \vec{e}_{far} \\ &= \vec{E}_{near} + \vec{E}_{far} + (\vec{e}_{near} - \vec{E}_{near}) \\ &= \vec{E} + (\vec{e}_{near} - \vec{E}_{near})\end{aligned}$$

Translation Table

Here	JDJ
$\vec{E}$	$\vec{E}$
$\vec{e}_{near}$	$\vec{E}_{near}$
$\vec{E}_{near}$	$\vec{E}_P$
$\vec{e}_{near} - \vec{E}_{near}$	$\vec{E}_i$

JDJ argues that $\vec{e}_{near} = 0$. If so,

$$\begin{aligned}\vec{e} &= \vec{E} - \vec{E}_{near} \\ &= \vec{E} + \frac{\vec{P}}{3\varepsilon_0}.\end{aligned}$$

Hence,

$$\vec{p} = \gamma \varepsilon_0 \vec{E} + \tfrac{1}{3}\gamma \vec{P}$$

$$\vec{P} = N\vec{p} = N\gamma\varepsilon_0 \vec{E} + \frac{N\gamma}{3}\vec{P},$$

$$\vec{P}\left(1 - \frac{N\gamma}{3}\right) = N\gamma\varepsilon_0\vec{E}$$

makes a prediction about density dependence of χ_e

$$\vec{P} = \varepsilon_0 \frac{N\gamma}{1 - \frac{N\gamma}{3}} \vec{E} \quad \Rightarrow \quad \boxed{\chi_e = \frac{N\gamma}{1 - \frac{N\gamma}{3}}}$$

Clausius Mossotti Eqn.

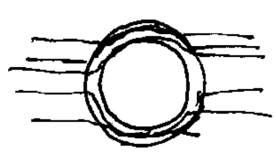
Comments about bdry value problems in magnetostatics.

$$\left.\begin{aligned}\nabla\cdot\vec{B} &= 0 \\ \nabla\times\vec{H} &= \vec{J}_f\end{aligned}\right\} \qquad \vec{B} = \mu\vec{J} \quad (\text{if linear}).$$

Comment on μ, linearity, "hard" magnetization, etc.

$$\vec{B} = \nabla\times\vec{A} \qquad \nabla^2\vec{A} = -\mu_0\vec{J}.$$

If $\vec{J}_f = 0$, $\vec{H} = -\nabla\Phi_H$, if linear, μ=const, $\nabla\cdot\vec{H} = 0$, $\nabla^2\Phi_H = 0$.

Mention problem of magnetic shielding. 

Mon (5)
9/16/02

Now Faraday's law.

J. starts with experimental evidence, EMF $\propto \frac{dF}{dt}$, uses Galilean invariance to find const k, then derives Faraday's law, or equ.

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (\mathrm{SI}).$$

We will just accept Faraday's equ, explore consequences. Faraday's equ. is actually simpler than expts that led to it.

Let C = closed curve in space, stationary for now. Need not be a loop of wire (just an imaginary curve maybe). Then Stoke's thm. says,

$$\oint_C \vec{E}\cdot d\vec{l} = -\int_{\substack{\text{surface}\\ S}} \frac{\partial \vec{B}}{\partial t}\cdot d\vec{a} = -\frac{dF}{dt}$$

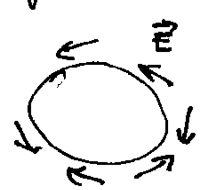
where $F = \int_S \vec{B}\cdot d\vec{a}$ = flux.

Here $\oint_C \vec{E}\cdot d\vec{l}$ = EMF, has dimensions of $\frac{\text{energy}}{\text{chg.}}$ = Voltage.

Things to notice:

~~(ii) $\oint \vec{E}\cdot d\vec{l}$ is taken at a fixed~~

① EMF = $\oint \vec{E}\cdot d\vec{l}$ only gives integral of $\vec{E}$, not $\vec{E}(\vec{x})$ along the curve. In fact, $\vec{E}$ can point in different direc's along C,



maybe

If you want $\vec{E}$ everywhere in space, have to solve full PDE's with bdry conds.

(2) $\text{EMF} = \oint \vec{E}\cdot d\vec{l}$ does not necessarily give $\frac{\text{energy}}{\text{chg}}$ that a charge would gain on going around loop, because charge would take time to go around, and integral is taken at fixed time (so if $\vec{E}$ = fn of t, answers would be different).

But if we assume (or at least $I = I(t)$, indep. of posn on loop.) that loop C coincides with a wire carrying a steady current I, then we can compute rate at which electric field does work on current in wire,

$$\frac{dW}{dt} = I \oint \vec{E}\cdot d\vec{l}$$

where integral is taken at the fixed time, so this is $-I\,\frac{dF}{dt}$.

The negative of this is the $\frac{\text{energy}}{\text{time}}$ that the battery (or whatever) must do work to maintain current I against $\vec{E}$.

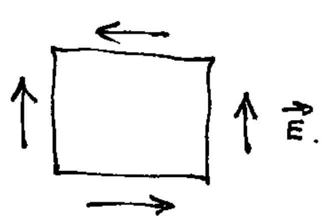
(3) If $\nabla\times\vec{E} \neq 0$, then $\vec{E}$ cannot be written as $\nabla\Phi$. However, since $\vec{B} = \nabla\times\vec{A}$, we can write

$$\nabla\times\left(\vec{E} + \frac{\partial\vec{A}}{\partial t}\right) = 0 \quad \Rightarrow \quad \exists\ \Phi \text{ such that}$$

$$\vec{E} + \frac{\partial\vec{A}}{\partial t} = -\nabla\Phi$$

or, $\boxed{\vec{E} = -\nabla\Phi - \frac{\partial\vec{A}}{\partial t}.}$ (neither Φ nor $\vec{A}$ unique, and Φ no longer has meaning of $-\int\vec{E}\cdot d\vec{l}$.)

(4) A question: If $\vec{E}$ points in different directions along a loop of wire, why does current flow? Or does it?



EMF $\neq 0$.