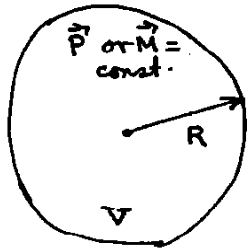
Fri. 9/12/02

Uniformly polarized or magnetized spheres.

(Useful model for particles eg proton with moments).



$V = \text{vol.} = \frac{4}{3}\pi R^3$

$\vec{p} = V\,\vec{P}$

$\vec{m} = V\,\vec{M}$

$$\Phi(\vec{x}) = \frac{1}{4\pi\epsilon_0}\int_V dv' \; \overset{\text{const}}{\vec{P}\cdot}\, \frac{\vec{x}-\vec{x}'}{|\vec{x}-\vec{x}'|^3} = \frac{1}{4\pi\epsilon_0}\vec{P}\cdot\left\{-\nabla\int_V \frac{dv'}{|\vec{x}-\vec{x}'|}\right\}$$

$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi}\int_V dv'\,\vec{M}\times\left(\frac{\vec{x}-\vec{x}'}{|\vec{x}-\vec{x}'|^3}\right) = \frac{\mu_0}{4\pi}\vec{M}\times\left\{-\nabla\int_V \text{same integral.}\right\}$$

$\{\ \} =$ $4\pi\epsilon_0 \times$ electric field produced by uniform $\rho = 1$ inside sphere.

(Can find by Gauss' law.)

$$= \begin{cases} V\,\dfrac{\vec{x}}{r^3}, & r > R \\[2ex] V\left(\dfrac{r}{R}\right)^3\dfrac{\vec{x}}{r^3} = V\,\dfrac{\vec{x}}{R^3}, & r < R. \end{cases}$$

So, exterior case: $(r > R)$.

$$\left.\begin{aligned} \Phi(\vec{x}) &= \frac{1}{4\pi\epsilon_0}\,\vec{p}\cdot\left(\frac{\vec{x}}{r^3}\right) \\ \vec{A}(\vec{x}) &= \frac{\mu_0}{4\pi}\,\vec{m}\times\left(\frac{\vec{x}}{r^3}\right) \end{aligned}\right\}$$ precisely dipole fields. (no higher moments).

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Compute $\vec{E}, \vec{B}$:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{3\vec{x}(\vec{x}\cdot\vec{p}) - r^2\vec{p}}{r^5}\right] = -\nabla\Phi$$

$$\vec{B} = \frac{\mu_0}{4\pi}\left[\frac{3\vec{x}(\vec{x}\cdot\vec{m}) - r^2\vec{m}}{r^5}\right] = \nabla\times\vec{A}$$

exactly same form.

Now internal field $(r<R)$. Express in terms of $\vec{P}, \vec{M}$ first:

$$\Phi(\vec{x}) = \frac{1}{4\pi\varepsilon_0}\frac{V}{R^3}\vec{P}\cdot\vec{x} = \frac{1}{3\varepsilon_0}\vec{P}\cdot\vec{x} = \frac{1}{4\pi\varepsilon_0}\frac{1}{R^3}\vec{p}\cdot\vec{x}$$

$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi}\frac{V}{R^3}\vec{M}\times\vec{x} = \frac{\mu_0}{3}\vec{M}\times\vec{x} = \frac{\mu_0}{4\pi}\frac{1}{R^3}\vec{m}\times\vec{x}.$$

<u>then find</u>

$$\vec{E} = -\nabla\Phi = -\frac{\vec{P}}{3\varepsilon_0} = -\frac{1}{4\pi\varepsilon_0}\frac{1}{R^3}\vec{p}$$

$$\vec{B} = \nabla\times\vec{A} = \frac{2}{3}\mu_0\vec{M} = \frac{\mu_0}{4\pi}\frac{1}{R^3}\cdot 2\vec{m}$$

Interior soln's are const, discontinuity at surface.

For use with elem. particles, want to take limit $R\to 0$ s.t. $\vec{p}$ or $\vec{m}$ = const.

Let $$f(r) = \begin{cases} \frac{1}{R^3}, & r<R \\ 0, & r>R \end{cases}. \qquad \int d^3\vec{x}\, f(r) = \frac{4\pi}{3}$$

$$\lim_{R\to 0} f(r) = \frac{4\pi}{3}\delta(\vec{r}).$$

So, interior solns as

$$R\to 0: \quad \left.\begin{aligned} \vec{E} &= -\frac{1}{3\varepsilon_0}\vec{p}\,\delta(\vec{x}) \\ \vec{B} &= +\frac{2}{3}\mu_0\vec{m}\,\delta(\vec{x}) \end{aligned}\right\}$$

responsible for "contact" terms in hyperfine structure etc.

b3h.