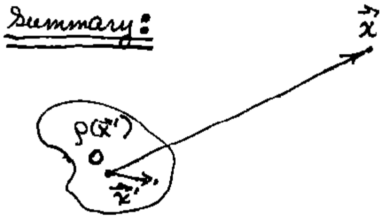
Summary:

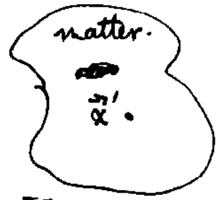


$$\Phi(\vec{x}) = \frac{1}{4\pi\varepsilon_0}\left\{ \frac{q}{r} + \vec{p}\cdot\frac{\vec{x}}{r^3} + \frac{1}{2} Q_{ij}\,\frac{x_i x_j}{r^5} + \dots \right\}$$

$$\left.\begin{aligned} q &= \int dv'\ \rho(\vec{x}\,') \\ \vec{p} &= \int dv'\ \vec{x}\,'\rho(\vec{x}\,') \\ Q_{ij} &= \int dv'\ (3x_i'x_j' - r'^2\delta_{ij})\,\rho(\vec{x}\,') \end{aligned}\right\} \text{same defs SI, Gaussian.}$$

Today want to talk about dielectrics. Dielectric means an insulator, contrast with conductor. no sharp distinction, but conductivity varies over many orders of magnitude.

So an ~~insulator~~ dielectric consists of molecules that contain bound charges, they are neutral so no monopole term, ~~but they~~ so dipole term dominate at large dist. Think of:



V = source vol.

$\cdot\,\vec{x}$ field point outside matter.

Then quadrupole term negligable, and reasonable to treat dipoles as a continuum.

This continuum model of dipoles very useful, much like contin. model of charge.

Define $\vec{P}(\vec{x}) = \dfrac{\text{dipole mom.}}{\text{vol.}} = \text{polarization}$ (same SI, Gauss.)

This is an example of a macroscopic average quantity ... give rough definition. Not defined on atomic scale.

Using expr. above for Φ, we now have ...

$$\Phi(\vec{x}) = \frac{1}{4\pi\varepsilon_0}\int_V dv'\ \vec{P}(\vec{x}\,')\cdot\frac{(\vec{x}-\vec{x}\,')}{|\vec{x}-\vec{x}\,'|^3} = \text{can transform by simple integration by parts...}$$

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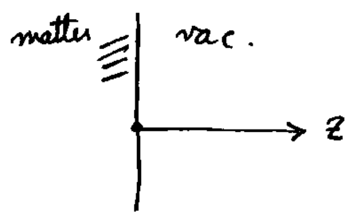
$$\Phi(\vec{x}) = \frac{1}{4\pi\varepsilon_0}\int_V dv' \frac{\rho_b(\vec{x}')}{|\vec{x}-\vec{x}'|} + \frac{1}{4\pi\varepsilon_0}\int_S da' \frac{\sigma_b(\vec{x}')}{|\vec{x}-\vec{x}'|}$$

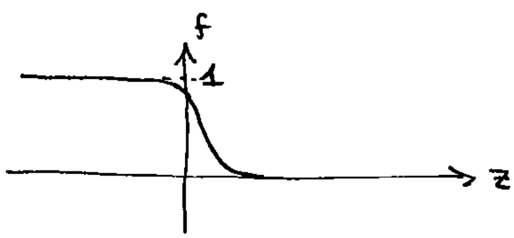
where
$$\left.\begin{aligned} \rho_b &= -\nabla\cdot\vec{P} \\ \sigma_b &= \hat{n}\cdot\vec{P} \end{aligned}\right\}$$
assume you are familiar with physics of these.

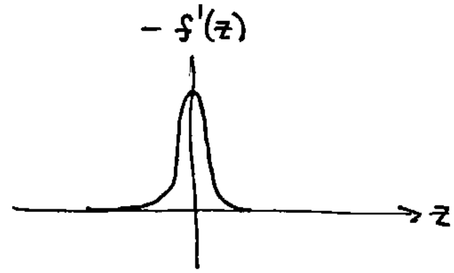
Jackson doesn't talk about σ_b, but can be regarded a special case of ρ_b.



Suppose density of matter falls off rapidly but continuously at surface.

$$\vec{P}(\vec{x}) = \vec{P}_0\, f(z).$$





$$\text{Then} \quad -\nabla\cdot\vec{P} = -f'(z)\,P_{0z}$$
$$= +\,\delta(z)\,P_{0z}.$$

$$\Rightarrow \quad \rho_b = \delta(z)\,\hat{n}\cdot\vec{P}$$
$$\sigma_b = \hat{n}\cdot\vec{P}.$$

Ok, so we can include σ_b with ρ_b if we want.

Can calculate $\vec{E}$ from Φ above ... can compute $\vec{E}$ due to bound (polarization) charges. But only valid when field pt exterior to bulk matter.

What if you are <u>inside</u> bulk matter? Then won't work obviously.

Turns out, however, that if you consider avg value of $\vec{E}$, then can still calculate from avg. bound charge distn, or from polarization. That is,

$$\varepsilon_0 \nabla \cdot \vec{E} = \rho_b (+ \rho_f \text{ if any}).$$

$$= \rho_f - \nabla \cdot \vec{P}$$

where now $\vec{E}$ is the macroscopic average $\vec{E}$ not the microscopic. Warning: J. doesn't distinguish notationally. Anyway, leads to defn,

$$\boxed{\vec{D} \equiv \varepsilon_0 \vec{E} + \vec{P}} \quad \text{electric displacement.}$$

$$\boxed{\nabla \cdot \vec{D} = \rho_f} \quad \text{one of macroscopic Maxwell eqns.}$$

Now, want to comment about linear, nonlinear media.

(1). As you know, in a linear medium, $\vec{P} = \varepsilon_0 \chi_e \vec{E}$, χ_e = susceptibility, hence $\vec{D} = \varepsilon_0 (1+\chi_e) \vec{E} = \varepsilon \vec{E}$, $\varepsilon \equiv \varepsilon_0 (1+\chi_e)$.

(2) Emphasize, relations above (defn of $\vec{P}$, $\vec{D}$, Maxwell eqn) do not depend on linearity.

(3) Elaborate, what does linearity mean? Later we will talk about χ_e as not a const., but fn. of ω. Will also hear, fn. of $\omega, \vec{k}$. May also hear, it's a fn of $\vec{x}$ or $(\vec{x}, t)$, as when medium is spatially dependent. So what does this mean?

Analogy with quantum mechanics (QM formalism helps).

		Dirac notation	
2 wave fns	$\psi(x)$	$\lvert\psi\rangle$	$\psi(x) = \langle x \lvert \psi\rangle$
	$\phi(x)$	$\lvert\phi\rangle$	$\phi(x) = \langle x \lvert \phi\rangle$

related by a linear operator A: $\lvert\phi\rangle = A\lvert\psi\rangle$.

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So, $\phi(x) = \langle x|\phi\rangle = \underbrace{\langle x|A|\psi\rangle}_{\boxed{\int dx'\,|x'\rangle\langle x'|}} = \int dx'\, \langle x|A|x'\rangle\langle x'|\psi\rangle$

insert resolution of identity, $= \int dx'\, A(x,x')\, \psi(x')$.

So general linear relationship has the form,

$$\phi(x) = \int dx'\, \underbrace{A(x,x')}_{}\, \psi(x') \qquad \phi = A\psi \text{ for short.}$$

kernel of A $= \langle x|A|x'\rangle$ (or x-space kernel).

~~Now what about case~~ So, to go back to $\vec{P}$ and $\vec{E}$, the general linear relation between them is

$$P_i(\vec{x}) = \varepsilon_0 \int dv' \quad \chi_{e\,ij}(\vec{x},\vec{x}')\; E_j(\vec{x}')$$

Have to make χ_e a tensor because $\vec{P}$ and $\vec{E}$ are vectors, and make it a fn. of 2 points because it's the kernel of a linear operator.

(Actually, this is <u>not</u> the most general linear relationship betw. $\vec{P}$ and $\vec{E}$, because we are ignoring t-dep, assuming everything is static and P determined solely by current value of $\vec{E}$. Later we worry about t-dep.)

Now make specializing assumptions.

1. Uniform medium $\Rightarrow$ Kernel $(\vec{x},\vec{x}')$ depends only on $\vec{x}-\vec{x}'$. In this case, $\chi_{e\,ij}(\vec{x}-\vec{x}')$

2. Isotropic medium means $\chi_{e\,ij}(\vec{x}-\vec{x}') = \chi_e(\vec{x}-\vec{x}')\,\delta_{ij}$ (Hence $\vec{P}$, $\vec{E}$ in same direction.

With these assumptions we have...

$$\vec{P}(\vec{x}) = \varepsilon_0 \int d\tau' \quad \chi_e(\vec{x}-\vec{x}') \ \vec{E}(\vec{x}').$$

③ Further assumption, $\chi_e(\vec{x}-\vec{x}') = \underset{\substack{\uparrow \\ \text{a const.}}}{\chi_e} \, \delta(\vec{x}-\vec{x}')$

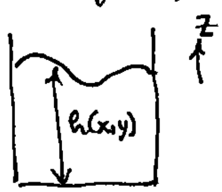
and you get $\vec{P} = \chi_e \vec{E}$ in sense of a dielectric const that is really const.

(Later we will discuss what it means to have a ω or $\vec{k}$ dependence).

Now some comments about the general, nonlinear case. Then we say,

$$\vec{P} = \vec{P}[\vec{E}] \qquad (\vec{P} \text{ is a \underline{functional} of } \vec{E}).$$

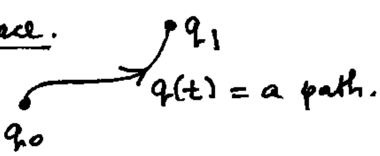
Concept of functional, eg. potential energy of a fluid is functional of surface



$$V = V[h(x,y)] \text{ or } V[h].$$

This is example of functional of single value. Another example, action functional in classical mechanics...

<u>Config. Space.</u>



$$A[q(t)] = \int_{t_0}^{t_1} L(q,\dot{q})\, dt = \text{action}.$$

$q(t)$ need not be physical path.

Here when we say $\vec{P}$ is fn'al of $\vec{E}$, we mean that $\vec{P}$ at any pt $\vec{x}$ is (in general) determined by values of $\vec{E}$ at <u>all</u> x points (not nec. local.)

Now, how do you get linear relations out of arb. (generally nonlinear) functional? A: Functional Taylor series.

ord. Taylor series $\quad f(x) = \text{const} + \text{linear term} + \text{quadr. term} \ldots$

So in present case it means,

$$\cancel{\vec{P}(\vec{x}) = \vec{P}_0(\vec{x}) + \varepsilon_0 \int d\vec{x}' \; \overleftrightarrow{\chi}_e(\vec{x},\vec{x}') \, \vec{E}(\vec{x}')}$$

$$P_i(\vec{x}) = \vec{P}_{0i}(\vec{x}) + \varepsilon_0 \int dv' \;\; \chi_{e\,ij}^{(1)}(\vec{x},\vec{x}') \; E_j(\vec{x}')$$

$$+ \frac{1}{2}\varepsilon_0 \int dv' dv'' \;\; \chi_{e\,ijk}^{(2)}(\vec{x},\vec{x}',\vec{x}'') \; E_j(\vec{x}') \, E_k(\vec{x}'') + \dots$$

$\vec{P}_0$ = polarizn in absence of $\vec{E}$

$\chi_{ij}^{(1)}(\vec{x},\vec{x}')$ = usual linear susceptibility

$\chi_{ijk}^{(2)}$ = quadratic suscept. etc.

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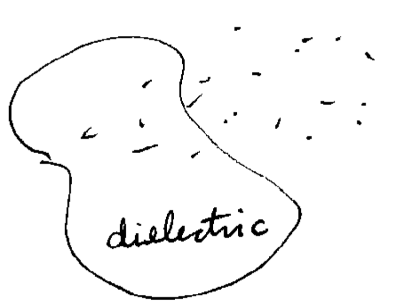
ok, you can't determine relation $\vec{P}[\vec{E}]$ unless you have some microscopic model.

Now, energy in presence of dielectrics. If no dielectrics around, then you know,

$$W = \frac{1}{2}\int dv \;\; \rho \Phi \;\; = \;\; \frac{\epsilon_0}{2}\int dv \;\; E^2.$$

What if there are dielectrics? Begin with general, nonlinear case. Then its hard to say what W is, in general. But can make simple stmts about increment in W when you change free charges by small amt.



free charges
ρ_f

some $\vec{E}, \vec{D}, \Phi$ exists.

Now bring in small increment
$\delta\rho_f = \delta\rho_f(\vec{x})$.

bzh approx.

Work required to do this is

$$\delta W = \int dv \;\; \delta\rho_f(\vec{x}) \, \Phi(\vec{x})$$