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Now return to Jackson's harmonic oscillator model of the dielectric fn.
Recall this gives,

N = # atoms/vol
Z = # electrons/atom
f_i = oscillator strengths

$$\epsilon = \epsilon_0 \left\{ 1 + \frac{Ne^2}{\epsilon_0 m} \sum_j \frac{f_j}{\omega_j^2 - \omega^2 - i\omega\gamma_j} \right\}, \qquad \sum_j f_j = Z.$$

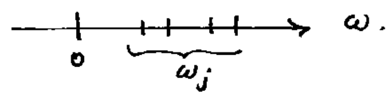
The model was crude but if resonant freq's ω_j and damping consts γ_j are understood in proper quantum mech'l sense the model is not bad for qualitative and (some) quantitative conclusions. First, a defn:

This was for a dielectric, the HO model involved only bound electrons.

$$\omega_p^2 = \frac{Ne^2 Z}{\epsilon_0 m} = (\text{plasma freq.})^2.$$

Resonant ω_j's are distributed along ω-axis and normally $\gamma_j \ll \omega_j$.

For an insulator (dielectric) the ω_j are nonzero.



Then note:

① $\text{Im}\,\epsilon$ small except when $|\omega - \omega_j| \sim \gamma_j$ for some j.

② as $\omega \to 0$, ϵ becomes real, and in fact

$$\epsilon(\omega=0) = \epsilon_0 \left\{ 1 + \frac{\omega_p^2}{Z} \sum_j \frac{f_j}{\omega_j^2} \right\} > \epsilon_0, \qquad \frac{\epsilon}{\epsilon_0} > 1.$$

For example, in H_2O $\frac{\epsilon}{\epsilon_0} \approx 10$ @ $\omega = 0$.

③ as $\omega \to \infty$, i.e., $\omega \gg$ any ω_j

$$\frac{\epsilon(\omega)}{\epsilon_0} \to 1 - \frac{\omega_p^2}{\omega^2} < 1.$$

④ So between $\omega = 0$ and $\omega \to \infty$, $\text{Re}\,\epsilon(\omega)$ must decrease from > 1 to < 1. But it doesn't do so monotonically.

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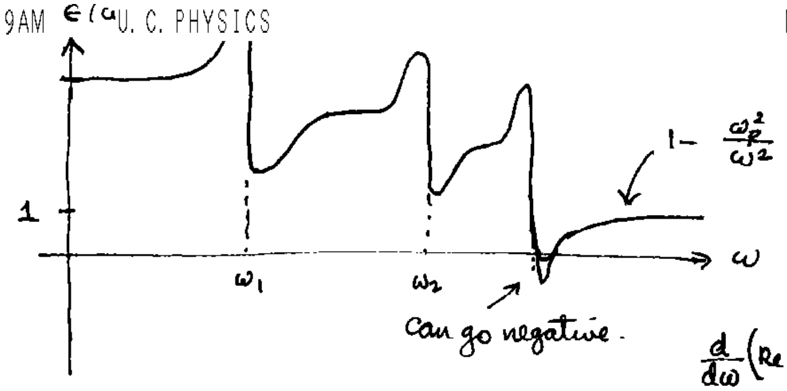
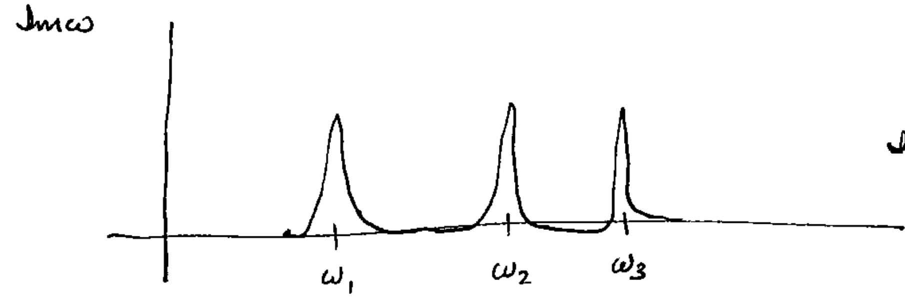


$\frac{d}{d\omega}\left(\mathrm{Re}\,\epsilon(\omega)\right) > 0$ most places but < 0 in resonance zone.



$\mathrm{Im}\,\epsilon > 0$

something like this, resonance model.

This is for a dielectric. For a <u>conductor</u>, we have "free" charges and currents, produced by unbound electrons. We also have

$$\vec{J}_f(\vec{x},\omega) = \sigma(\omega)\,\vec{E}(\vec{x},\omega).$$

Conductor also has bound charges that can create polarization $\vec{P}$, usually don't worry about this at $\omega = 0$, but for $\omega > 0$ it's important too.

Look at ~~[illegible]~~ ω-domain Maxwell eqns for conductor, ~~[illegible]~~ all fields are fns. of $(\vec{x},\omega)$, $\vec{D} = \epsilon(\omega)\,\vec{E}$, usual defn. of ϵ,

$$\epsilon(\omega) = \epsilon_0\left(1 + \chi_e(\omega)\right)$$

↑ describes polarization, $\vec{P} = \epsilon_0 \chi_e \vec{E}$.

(Assume $\mu = \mu_0$).

$$\nabla\cdot(\epsilon\vec{E}) = \nabla\cdot\vec{D} = \rho_f$$
$$\nabla\cdot\vec{B} = 0$$
$$\nabla\times\vec{E} = i\omega\vec{B} = i\omega\mu_0\vec{H}$$
$$\nabla\times\vec{H} = \vec{J}_f - i\omega\vec{D} = \sigma\vec{E} - i\omega\epsilon\vec{E}. = (\sigma - i\omega\epsilon)\vec{E}.$$

in Amperes law, look at RHS

The ϵ incorporates effects of bound electrons, σ of conduction electrons.

Makes it logical to rewrite this,

$$\epsilon \rightarrow \epsilon_b = \epsilon \text{ due to bound electrons.}$$

$$\epsilon_{tot} = \epsilon_b + \frac{i\sigma}{\omega}.$$

Then $\nabla \times \vec{H} = -i\omega\,\epsilon_{tot}\,\vec{E}$.

As for Gauss, $\nabla \cdot \vec{D} = \nabla \cdot (\epsilon_b \vec{E}) = \rho_f$.

But continuity eqn for free charges is

$$\underbrace{\nabla \cdot \vec{J}_f}_{\downarrow} - i\omega\,\rho_f = 0$$

$$\nabla \cdot (\sigma \vec{E}), \quad \text{or} \quad \rho_f = \frac{-i\sigma}{\omega}\nabla \cdot \vec{E}.$$

So, can write Gauss' law as $\nabla \cdot \left(\epsilon_b + \frac{i\sigma}{\omega}\right)\vec{E} = \nabla \cdot \left(\epsilon_{tot}\vec{E}\right) = 0.$

Thus, Maxwell eqns become

$$\boxed{\epsilon_{tot} = \epsilon_b + \frac{i\sigma}{\omega}}$$

$$\left.\begin{aligned} \nabla \cdot (\epsilon_{tot}\vec{E}) &= 0 \\ \nabla \times \vec{H} &= -i\omega\,\epsilon_{tot}\,\vec{E} \end{aligned}\right\} + \text{homog. eqns.}$$

Same as in a dielectric with $\epsilon \rightarrow \epsilon_{tot}$.

Thus with this change we can treat dielectrics and conductors on an equal footing.

Can also incorporate conduction electrons into resonance model.

Recall, driven, damped HO model:

$$m\left(\ddot{\vec{\xi}} + \gamma_0\dot{\vec{\xi}} + \omega_0^2\vec{\xi}\right) = -e\vec{E}(\vec{x},t).$$

If $\omega_0 = 0$, its a free particle with friction. γ_0 can be thought of as a collision time.

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So leads to a term in the resonance model of the dielectric function with $\omega_0 = 0$, γ_0 = some damping.

$$\epsilon_{tot} = \epsilon_0 + \frac{Ne^2}{m} \frac{f_0}{-\omega^2 - i\omega\gamma_0} + \cancel{\sum_{j=0}}$$

So conductivity comes from a resonance at $\omega = 0$

conduction elec's. ($j=0$ term)

$$+ \frac{Ne^2}{m} \sum_{j \neq 0} \frac{f_j}{\omega_j^2 - \omega^2 - i\omega\gamma_j}$$

bound elec's.

For insulators all $\omega_j > 0$, but for conductors, one $\omega_0 = 0$. ($j=0$).

Setting the conduction term to $\frac{i\sigma}{\omega}$, we get

$$\boxed{\sigma = \frac{Ne^2}{m} \frac{f_0}{\gamma_0 - i\omega}}$$

crude, qualitatively correct.

$f_0 = Z \times$ frac. of electrons that are conduction electrons.

This model indicates that when $\omega \ll \gamma_0$, conductivity is <u>real</u>.

Now, dispersion relation from M.E.'s is

also if $\omega \ll \omega_j$, then

$$\left| \frac{f_0}{-i\omega\gamma_0} \right| \ll \left| \frac{f_j}{\omega_j^2 - \omega^2 - i\omega\gamma_j} \right|$$

$$k^2 = \omega^2 \epsilon_{tot}(\omega) \mu_0 .$$

since $\mu = \mu_0$ here

dominates at low ω

$$k^2 = \frac{\omega^2}{c^2} \frac{\epsilon_{tot}(\omega)}{\epsilon_0} = \frac{\omega^2}{c^2} \left(\frac{\epsilon_b(\omega)}{\epsilon_0} + \frac{i\sigma}{\epsilon_0 \omega} \right).$$

$$k^2 = \frac{i\sigma\omega}{c^2\epsilon_0} = i\sigma\mu_0\omega ,$$

$$k = \left(\frac{1 \pm i}{\sqrt{2}}\right) \sqrt{\sigma\mu_0\omega} , \qquad e^{ikz} \sim e^{-z/\delta} , \qquad \delta = \sqrt{\frac{2}{\sigma\mu\omega}} = \text{skin depth.}$$