$$\vec{E} = e\,\frac{\hat{n}\times[(\hat{n}-\vec{\beta})\times\dot{\vec{\beta}}]}{cR\,(1-\hat{n}\cdot\vec{\beta})^3}\,, \qquad \vec{A}(t) = \sqrt{\frac{c}{4\pi}}\; R\vec{E}(t).$$

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$$\frac{d^2W}{d\omega d\Omega} = 2\,|\vec{A}(\omega)|^2$$

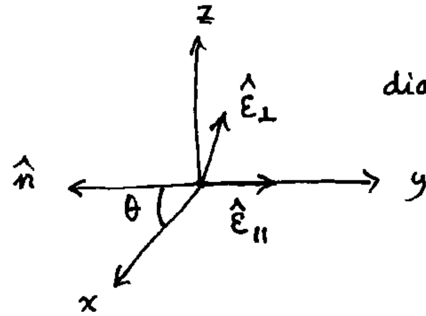


diagram for polarization vectors.

$$\vec{A}(\omega) = A_\perp(\omega)\,\hat{\varepsilon}_\perp + A_{\parallel}(\omega)\,\hat{\varepsilon}_{\parallel}.$$

$$\begin{Bmatrix} A_\perp(\omega) \\ A_{\parallel}(\omega) \end{Bmatrix} = \begin{pmatrix}\text{const}\\ \text{phase}\end{pmatrix}\sqrt{\frac{e^2}{8\pi^2 c}}\;\beta\omega \times \begin{Bmatrix} \sin\theta \displaystyle\int_{-\infty}^{+\infty} dt\; e^{i\varphi(t)}\cos\omega_0 t \\ \displaystyle\int_{-\infty}^{\infty} dt\; e^{i\varphi(t)}\sin\omega_0 t \end{Bmatrix}. \qquad \varphi(t) = \omega\left(t - \frac{\hat{n}\cdot\vec{r}(t)}{c}\right)$$

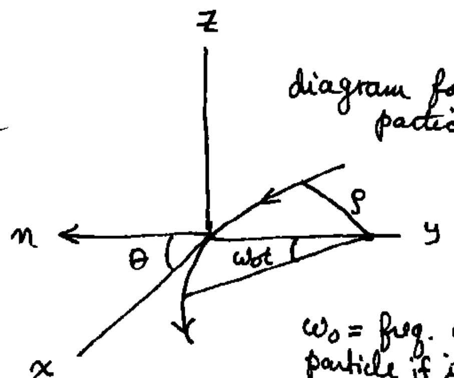


diagram for particle orbit.

ω_0 = freq. of motion of particle if it moved exactly in a circle.

<u>approx</u>: $\frac{1}{\gamma} \sim \theta \sim \omega_0 t$

$\cos\omega_0 t \to 1$

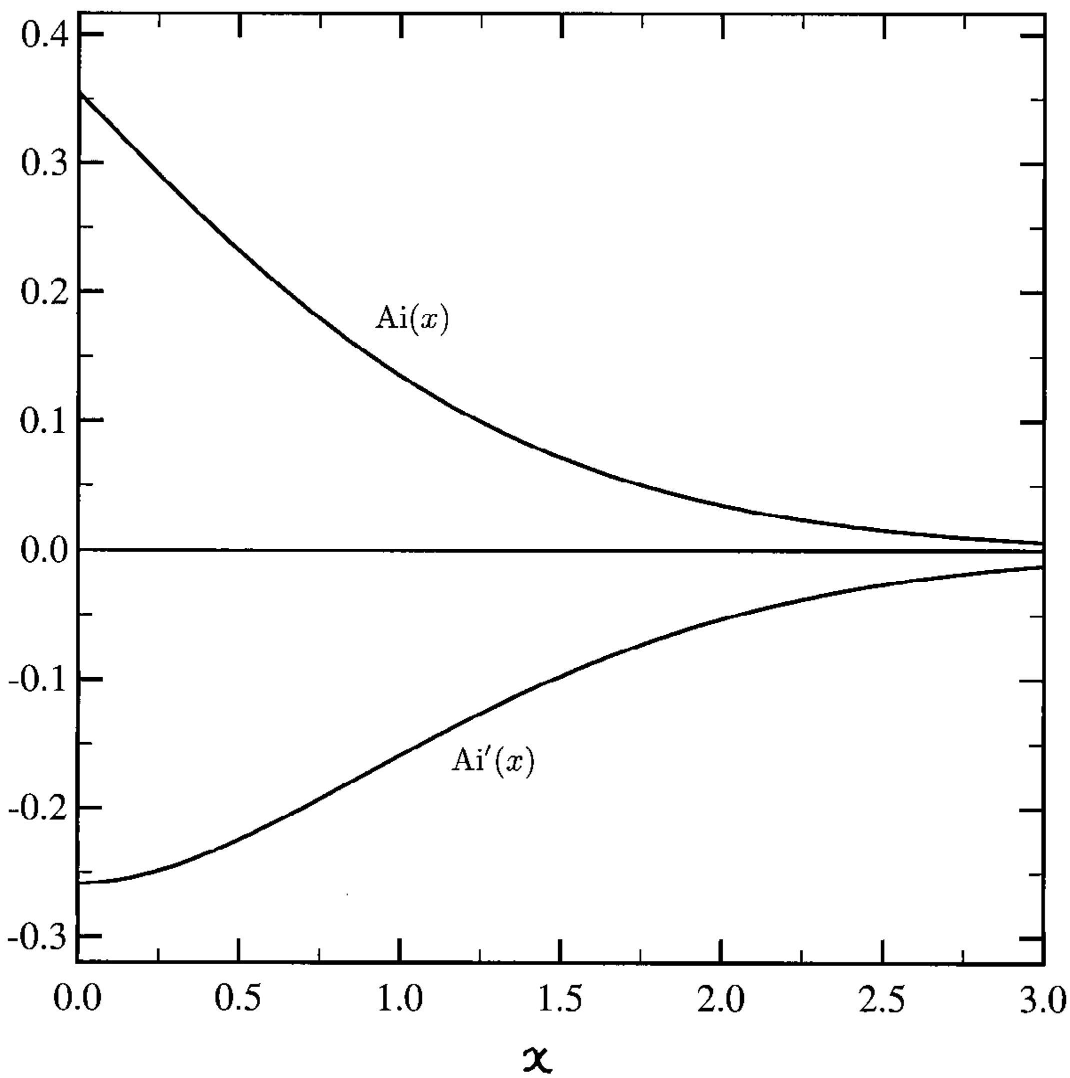
$\sin\omega_0 t \to \omega_0 t$

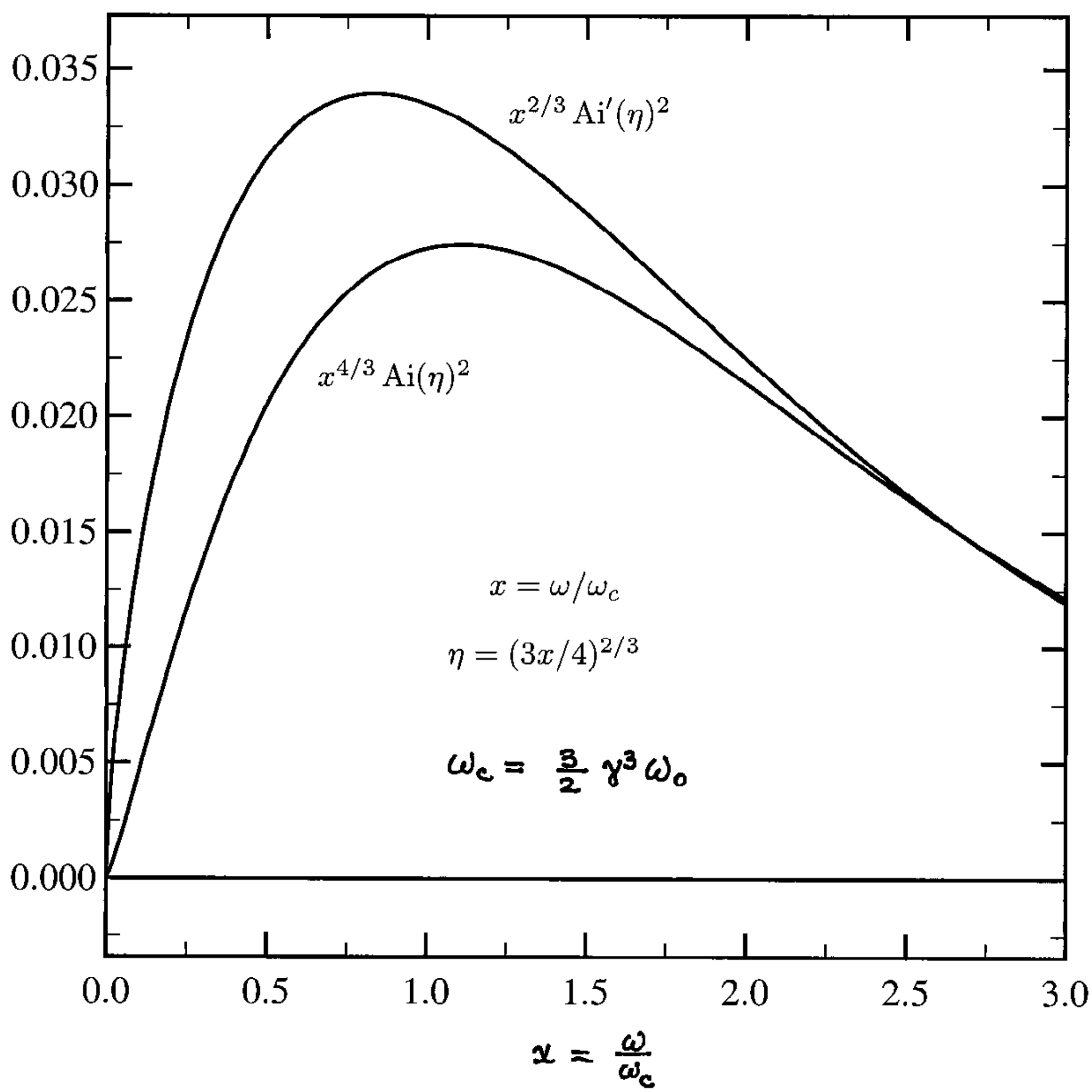
$\sin\theta \to \theta$

$$\varphi(t) \to \frac{\omega t}{2}\left(\frac{1}{\gamma^2}+\theta^2\right) + \frac{\omega\omega_0^2}{6}\,t^3.$$

$$Ai(\eta) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} dz\; e^{i(\eta z + z^3/3)}$$

$$Ai'(\eta) = \frac{i}{2\pi}\int_{-\infty}^{+\infty} dz\; z\, e^{i(\eta z + z^3/3)}$$


0.4
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
Ai(x)
Ai′(x)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
x


0.035
0.030
0.025
0.020
0.015
0.010
0.005
0.000
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$x^{2/3}\,\mathrm{Ai}'(\eta)^2$
$x^{4/3}\,\mathrm{Ai}(\eta)^2$
$x = \omega/\omega_c$
$\eta = (3x/4)^{2/3}$
$\omega_c = \frac{3}{2}\gamma^3\omega_0$
$x = \frac{\omega}{\omega_c}$

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Look at large, small η limits:

η small, $$Ai(\eta) = \frac{1}{3^{2/3}\,\Gamma(2/3)} + O(\eta)$$

$$Ai'(\eta) = \frac{-1}{3^{1/3}\,\Gamma(1/3)} + O(\eta^2).$$

η large, $$Ai(\eta) = \frac{e^{-\frac{2}{3}\eta^{3/2}}}{2\sqrt{\pi}\,\eta^{1/4}}$$

$$Ai'(\eta) = \frac{-\eta^{1/4}}{2\sqrt{\pi}}\, e^{-2/3\,\eta^{3/2}}$$

Gives limiting forms for ~~energy~~ power radiated,

$$\left.\begin{aligned}\left(\frac{d^2W}{d\omega\, d\Omega}\right)_\perp &= \frac{e^2}{c}\,\theta^2\left(\frac{2\omega^2}{\omega_0^2}\right)^{2/3}\frac{1}{3^{4/3}\,\Gamma(2/3)^2} \sim \omega^{4/3}\\ \left(\frac{d^2W}{d\omega\, d\Omega}\right)_\parallel &= \frac{e^2}{c}\left(\frac{4\omega}{\omega_0}\right)^{2/3}\frac{1}{3^{2/3}\,\Gamma(1/3)^2} \sim \omega^{2/3}\end{aligned}\right\}\ \eta \text{ small.}$$

$$\left.\begin{aligned}\left(\frac{d^2W}{d\omega\, d\Omega}\right)_\perp &= \frac{e^2}{c}\,\frac{\theta^2}{2\pi}\,\frac{\omega}{\omega_0}\,\frac{1}{\left(\frac{1}{\gamma^2}+\theta^2\right)^{1/2}}\, e^{-2\xi}\\ \left(\frac{d^2W}{d\omega\, d\Omega}\right)_\parallel &= \frac{e^2}{c}\,\frac{1}{2\pi}\,\frac{\omega}{\omega_0}\left(\frac{1}{\gamma^2}+\theta^2\right)^{1/2} e^{-2\xi}\end{aligned}\right\}\ \eta \text{ large.}$$

η large means ξ large means ω large. How large?

Set $\theta = 0$, then $\xi = \dfrac{\omega}{3\gamma^3\omega_0}$, exponent $= e^{-\omega/\omega_c}$

where $\boxed{\omega_c = \frac{3}{2}\gamma^3\omega_0}$ agrees with earlier estimates.

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Remarks about wigglers and undulators...

Now move to selected topics from Ch. 7. of the book. We begin with waves in media. Since we have a medium we need macroscopic Maxwell eqns, $\vec{D}$ and $\vec{H}$. We also return to SI units.

We assume a linear medium, so $\vec{D} = \epsilon\vec{E}$ and $\vec{B} = \mu\vec{H}$. Jackson in Sec. 7.1 starts by writing $\epsilon = \epsilon(\omega)$, but he doesn't explain what that means. So before getting into plane waves let's discuss meaning of ϵ, $\epsilon(\omega)$, $\epsilon(\vec{x},\omega)$, $\epsilon(\vec{k},\omega)$ etc etc.

Back in electrostatics we explained that ϵ is really an <u>operator</u>, that maps $\vec{E}(x)$ into $\vec{D}(\vec{x})$. Also explained that most general form of a linear operator is an integral transform,

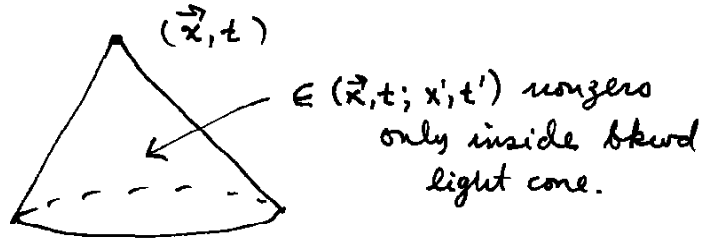
$$D_i(\vec{x}) = \sum_j \int d^3\vec{x}'\ \epsilon_{ij}(\vec{x},\vec{x}')\ E_j(\vec{x}'),$$

where $\epsilon_{ij}(\vec{x},\vec{x}')$ is the <u>kernel</u> of the transform and it is a tensor in ij because in general $\vec{D}$ is not in the same direction as $\vec{E}$. This is just a stmt of linearity and has no physics in it. The functional form of $\epsilon_{ij}(\vec{x},\vec{x}')$ depends on the properties of the medium and can't be determined without some microscopic model for the medium. This eqn says that the value of $\vec{D}$ at one spatial point $\vec{x}$ depends in general on the values of $\vec{E}$ at all other spatial points, that is, that it is <u>nonlocal</u>. Later we will specialize to local models.

For the general t-dep. case we must write

$$D_i(\vec{x},t) = \sum_j \int d^3\vec{x}' \int dt'\ \ \epsilon_{ij}(\vec{x},t;\vec{x}',t')\ E_j(\vec{x}',t')$$

But the value of $\vec{D}$ at $(\vec{x},t)$ doesn't really depend on $\vec{E}$ at all other $(\vec{x}',t')$, because of <u>causality</u>. In fact, for given $(\vec{x},t)$, ~~$\epsilon_{ij}(\vec{x}',t')$~~ $\epsilon(\vec{x},t;\vec{x}',t')$ must vanish if $(\vec{x}',t')$ is outside the backward light cone from $(\vec{x},t)$.



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~~If we specialize to~~

There are various specializing assumptions we can make about the physics that lead to restrictions on the form of the kernel $\epsilon_{ij}(\vec{x},t;\vec{x}',t')$. Begin with the assumption of a <u>static</u> <u>medium</u>, one whose properties are time independent. Then if $\vec{E}(\vec{x},t)$ gives $\vec{D}(\vec{x},t)$ in the relation $\vec{D}=\epsilon\vec{E}$, then $\vec{E}(\vec{x},t+\tau)$ must give $\vec{D}(\vec{x},t+\tau)$. That is, we must have

$$D_i(\vec{x},t+\tau) = \sum_j \int d^3\vec{x}'\,dt' \quad \epsilon_{ij}(\vec{x}t;\vec{x}'t')\;E_j(\vec{x}',t'+\tau).$$

But by replacing $t\to t-\tau$, $t'\to t'-\tau$ we obtain

$$D_i(\vec{x},t) = \sum_j \int d^3\vec{x}'\,dt' \;\; \epsilon_{ij}(\vec{x},t-\tau;\vec{x}',t'-\tau)\,E_j(\vec{x}',t').$$

Since this must be true for all $E_j(\vec{x}',t')$, we obtain

$$\epsilon_{ij}(\vec{x},t-\tau;\vec{x}',t'-\tau) = \epsilon_{ij}(\vec{x},t;\vec{x}',t') \qquad \left(\begin{matrix}\text{static}\\ \text{medium}\end{matrix}\right).$$

This implies that ϵ_{ij} is really only a function of the time difference, $t-t'$:

$$\epsilon_{ij}(\vec{x},t;\vec{x}',t') = \epsilon_{ij}(\vec{x},\vec{x}';t-t').$$

Notice the abuse of notation; these are really 2 different functions.