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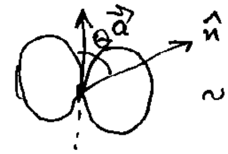
Summary.

$$\vec{E}_{radn} = e \frac{\hat{n} \times [(\hat{n} - \vec{\beta}) \times \dot{\vec{\beta}}]}{cR(1 - \hat{n}\cdot\vec{\beta})^3}$$

$$\vec{B}_{radn} = \hat{n} \times \vec{E}$$

$v \ll c$, $\quad \mathcal{P} = \frac{2}{3}\frac{e^2 a^2}{c^3}$ (Larmor)



$\sim \sin^2\theta$

any v, $\quad \frac{dE_{rad}}{dt'} = \frac{2}{3}\frac{e^2}{c}\gamma^6 \left[\dot{\beta}^2 - (\vec{\beta}\times\dot{\vec{\beta}})^2\right]$ $\qquad t'$ = particle (ret.) time.

$\boxed{\text{case } \vec{\beta} \parallel \dot{\vec{\beta}}}$ $= \frac{2}{3}\frac{e^2}{c^3}\gamma^6 a^2$

$$P' = \frac{dE_{rad}}{dt'} = \frac{2}{3}\frac{e^2}{m^2c^2}F^2 \cancel{= \frac{2}{3}\frac{e^2}{m^2c^4}\dot\beta}$$

$$\frac{\left(\frac{dE_{rad}}{dt'}\right)}{\left(\frac{dE}{dt'}\right)} = \frac{2}{3}\frac{e^2}{m^2c^4}\frac{1}{\beta}\frac{dE}{dx}. = \text{measure of radiation losses in } \parallel \text{ acceleration.}$$

If $\frac{\text{rad}}{\text{particle gain}} \sim 1$, then $\frac{dE}{dx} \sim \frac{mc^2}{10^{-13}\text{cm}}$.

This is hugely more than any $\frac{dE}{dx}$ that can be achieved technologically.

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$\Rightarrow$ Energy loss due to radiation <u>not</u> <u>important</u> in linear accelerators.

Now specialize to case $\vec{v} \perp \vec{a}$, as in circular accelerators.

$$\frac{dE_{rad}}{dt}\left(\text{means } \frac{dE}{dt_{ret}}\right) = \frac{2}{3}\frac{e^2}{c^3}\gamma^4 a^2$$

Express in terms of force, $F = m\gamma a$ for $\perp$ force as in circular motion $(\vec{F} \perp \vec{v})$. Thus

$$= \frac{2}{3}\frac{e^2}{c^3 m^2}\gamma^2 F^2$$

Factor of γ^2 larger than $\parallel$ accel. for same force.

Thus, if $F_\perp \sim F_\parallel$ and $\beta \to 1$ $(\gamma \gg 1)$, then <u>radiation due to $\perp$ acceleration dominates that due to $\parallel$ accel.</u>

For circular accelerators, $F = e\frac{v}{c}B$, $\rho = \frac{mc\gamma v}{eB}$ = radius, express $\frac{dE_{rad}}{dt}$ in terms of energy of particle,

$$\frac{dE_{rad}}{dt} = \frac{2}{3}\frac{e^2}{c^3}\beta^4\frac{E^4}{m^4}\frac{1}{\rho^2}.$$

So, for equal E, ρ, radiation losses are $\left(\frac{m_e}{m_p}\right)^4 \simeq 10^{13} \times$ worse for electrons than protons. Makes high energy electron synchrotrons difficult.

(2)

Now $\boxed{\text{Qualitative Features of Radiation Pattern}}$.

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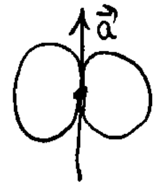
at low velocities, we have $\sin^2\theta$ pattern relative to $\vec{a}$:



and $\vec{v}$ doesn't matter.

at higher velocities, this gets blue-shifted (mostly) and aberrated due to motion of particle.

photons coming off.

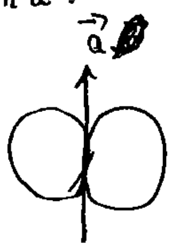
rest frame

photons aberrated forward.

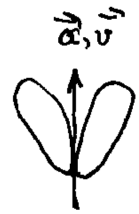
lab frame

Thus for $\vec{v} \parallel \vec{a}$:

low vel.

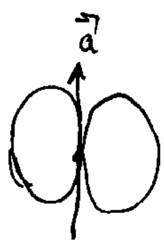


high vel $\longrightarrow$



and for $\vec{v} \perp \vec{a}$:

low vel



high vel $\longrightarrow$

So you get these lobes of radiation. Now... look at qualitative semi-quantitative features of angular dependence of intensity...

$$\underset{\text{field time}}{\rightarrow} \frac{d^2E}{dt\,d\Omega} = \frac{\text{energy radiated}}{\text{time - solid angle}} = R^2\, \vec{S}\cdot\hat{n} = \frac{c}{4\pi} R^2 E^2$$

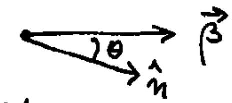
$$\text{or} \quad \frac{dE}{dt'\,d\Omega} = R^2 (1-\vec{\beta}\cdot\hat{n})\, \vec{S}\cdot\hat{n} = \frac{c}{4\pi} R^2 (1-\vec{\beta}\cdot\hat{n}) E^2.$$

particle (retarded) time.

~~The~~ Note, $E^2 = \dfrac{e^2}{c^2R^2}\,\dfrac{\left|\hat{n}\times[(\hat{n}-\vec{\beta})\times\dot{\vec{\beta}}]\right|^2}{(1-\hat{n}\cdot\vec{\beta})^6}$.

So you get $(1-\vec{\beta}\hat{n})^5$ or $(1-\vec{\beta}\cdot\hat{n})^6$ in denom, depending on which time you want to use. This factor controls the manner in which the radiation is thrown into forward lobes. Take ultra relativ. case $\gamma \gg 1$.

Write $1-\hat{n}\cdot\vec{\beta} = 1-\beta\cos\theta$,



(not same θ as above), this is $O(1)$ unless $\beta\cos\theta \approx 1$,

which can occur only when $\beta\approx 1$ and $\cos\theta\approx 1$, i.e. γ large and θ small. Then denom becomes small and radiation intensity magnified.

<u>Note:</u> $\gamma = \dfrac{1}{\sqrt{1-\beta^2}} \Rightarrow \beta^2 = 1-\dfrac{1}{\gamma^2}$, $\beta \approx 1-\dfrac{1}{2\gamma^2}$, if $\gamma\gg 1$.

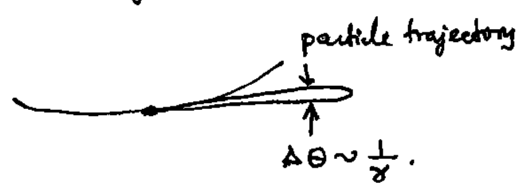
and $\cos\theta \approx 1-\dfrac{\theta^2}{2}$.

$$1-\beta\cos\theta = 1-\left(1-\frac{1}{2\gamma^2}\right)\left(1-\frac{\theta^2}{2}\right) = 1-1+\frac{1}{2\gamma^2}+\frac{\theta^2}{2} = \frac{1}{2\gamma^2}(1+\gamma^2\theta^2).$$

So for most angles, denom. gives a factor $\sim O(1)$, but when $\gamma\theta\sim 1$, i.e. $\theta \sim \frac{1}{\gamma}$, then denom. gives multiplication by

$$\sim (2\gamma^2)^5 \text{ or } (2\gamma^2)^6$$

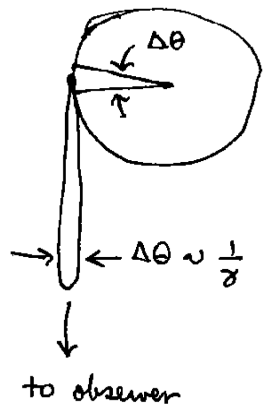
depending on which time you want to use. Huge magnification in fwd direction ("search light" effect).



(There are factors in the numerator that reduce the power of γ at max intensity, but the effect remains. You find max intensity $\sim \gamma^8$ for $\dot{\vec{\beta}} \parallel \vec{\beta}$, γ^6 for $\perp$ case.)
(This for const. $\vec{a}$).

Qualitative features of Radiation Power Spectrum at high γ (Synchrotron Radiation)

Take case of circular motion.



angular width of pulse is also angle of circle over which observer sees radiation.

ω = freq. of orbit
ρ = radius.
v = vel. of particle (sim. β, γ).

$$\Delta\tau = \text{time particle spends on arc } \Delta\theta = \frac{\rho\,\Delta\theta}{v}.$$

Suppose particle starts emitting photons at beg. of $\Delta\theta$ segment, and stops at end. (In reality it emits photons at other times too but only these reach the observer.) Thus particle emits for time $\Delta\tau$. Creates a pulse of light heading toward observer. In time $\Delta\tau$, head of light pulse travels dist $c\,\Delta\tau$, while particle moves $v\,\Delta\tau$. So len. of pulse is $(c-v)\Delta\tau$

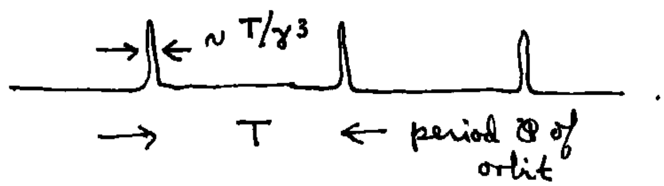
$$(c-v)\Delta\tau = (c-v)\frac{\Delta\theta}{v}\rho = \left(\frac{1}{\beta}-1\right)\Delta\theta\,\rho \sim \frac{\Delta\theta\,\rho}{\gamma^2}.$$

So time duration of pulse as seen by observer is this div. by c,

$$\left(\begin{matrix}\text{pulse width}\\ \text{in time}\end{matrix}\right) = \frac{\Delta\theta\,\rho}{c\,\gamma^2} \sim \frac{\rho}{c\gamma^3}. \qquad \text{since } \Delta\theta \sim \frac{1}{\gamma}$$

But period of motion is $\frac{2\pi\rho}{v} \sim \frac{\rho}{c}$, so $\frac{\tau_{pulse}}{\tau_{period}} \sim \frac{1}{\gamma^3}$.

As seen by observer:



In other words,

$$\omega_{max}\,(\text{light}) \sim \gamma^3 \omega_0 \quad \leftarrow \text{freq. of orbit.}$$

γ^3 may be big factor, e.g. 1 GeV electron has $\gamma = 2000$, $\gamma^3 \sim 10^{10}$.

<u>Power spectrum etc. of synchrotron radiation.</u>

Synchrotron radiation is produced by relativistic electrons $\gamma \gg 1$ in magnetic fields. Occurs commonly in astrophysics. To motivate following, suppose we are making measurements of synch. radn. and trying to interpret it. We can put filters in front of our detectors to select a polarization and a frequency (or small frequency interval), then we measure the power received.

The observer in this case (us) is far away from the source and the distance to the source is $\gg$ radii of orbits. The vector $\hat{n}$ is nearly const.

Consider a (radn produce by) single particle. Because of searchlight effect, we will only see radiation when particle's motion is nearly pointing directly at us. Then we see a pulse of light generated by a small segment of the particle's orbit. We can approximate this small segment by a circle of radius ρ, which is the radius of curvature of the small segment. Nothing says the particle will follow a circular orbit (in the large); this is just an approximation for a small segment.

Let W = energy received by some detector. Then

$$\frac{d^2W}{dt\,d\Omega} = R^2\,\vec{S}\cdot\hat{n} \qquad \text{But } \vec{S} = \frac{c}{4\pi}E^2\,\hat{n}, \text{ so}$$

$$= \frac{c}{4\pi}R^2E^2.$$

Here this is $\vec{E}(\vec{x},t)$, $\vec{x}$ = observer's posn = const in this discussion so just think of $\vec{E}(t)$.

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Integrate over time,

$$\frac{dW}{d\Omega} = \int_{-\infty}^{+\infty} dt \; \frac{c}{4\pi} R^2 |\vec{E}(t)|^2 . \quad = \int_{-\infty}^{+\infty} dt \, |\vec{A}(t)|^2$$

Jackson writes $\vec{A}(t) = \sqrt{\frac{c}{4\pi}} \, R \, \vec{E}(t)$, for convenience. So
(this is not the vector potential.)

Then uses Parseval's thm,

$$= \int_{-\infty}^{+\infty} d\omega \; |\vec{A}(\vec{\omega})|^2 \qquad \text{where} \quad \vec{A}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dt \; e^{i\omega t} \vec{A}(t).$$

Thus we strip off the $\int d\omega$, and define

$$\frac{d^2W}{d\omega d\Omega} = |\vec{A}(\omega)|^2 .$$

This is the <u>power spectrum</u>.
(but see below for factor of 2).

Note that $\frac{d^2W}{d\omega d\Omega}$ is <u>not</u> the Fourier transform of $\frac{d^2W}{dt d\Omega}$.

<u>Small point</u>: $\vec{A}(-\omega) = \vec{A}(\omega)^*$ since $\vec{A}(t) =$ real. So we fold negative frequencies over to pos. ones, write

$$\frac{dW}{d\Omega} = 2 \int_0^{\infty} d\omega \; |\vec{A}(\omega)|^2$$

hence $$\frac{d^2W}{d\omega d\Omega} = 2 \; |A(\vec{\omega})|^2 \qquad (\omega \geq 0 \text{ only now}).$$

So we need to find $\vec{A}(\omega)$.