

Physics 209
Fall 2002
Homework 7
Due Friday, October 18 at 5:00pm.

1. A relativistic rocket ship carries matter-antimatter fuel. The matter and antimatter are allowed to annihilate, and the energy is released as photons that are directed out the exhaust in the direction opposite to the desired acceleration of the rocket ship. As time progresses, the mass of the ship decreases, due to the consumption of the fuel.

The rate at which the fuel is consumed is not necessarily constant. For instance, it is desirable to maintain the acceleration in the rest frame of the rocket to be less than or equal to g , the acceleration of gravity at the surface of the earth, for the comfort of the passengers. Therefore, as the mass of the rocket ship decreases, the rate of fuel consumption must be reduced to keep the acceleration from exceeding g .

(a) Find a relation between m/m_0 and v , where m_0 is the initial mass of the rocket ship, m is the mass at some later time (or proper time) when the velocity is v .

(b) It is desired to make a journey to alpha Centauri (4 light years distant) and to return. The ship will accelerate with proper acceleration g for the first half of the trip to the star, then decelerate for the second half of the trip out. On return, the process is reversed. Calculate the ratio m_f/m_0 , where m_f is the final mass of the rocket ship on return to earth. This gives the ratio of fuel to payload for such a journey.

(c) Find the time by which the passengers will have aged on return to Earth, compared to people who stayed at home. Don't count the time spent exploring alpha Centauri.

2. It is often useful to think of a plane light wave as a beam of photons of constant density n (number of photons per unit volume) all moving in the same direction $\hat{\mathbf{k}}$. The photons have energy $\hbar\omega$, where ω is the frequency of the light wave.

(a) Let the photon beam be travelling in the direction $\hat{\mathbf{k}} = \hat{\mathbf{x}} \cos \theta + \hat{\mathbf{y}} \sin \theta$ in the unprimed ("stationary") frame. Let the primed ("moving") frame be moving with velocity v down the x -axis relative to the unprimed frame. Find the photon number density n' and direction $\hat{\mathbf{k}}'$ in the moving frame. Show that $\hat{\mathbf{k}}'$ is consistent with the relativistic laws of aberration.

(b) In the primed frame, the photons no longer have energy $\hbar\omega$, but rather $\hbar\omega'$. Let u, u' be the energy densities in the two frames. Find u' in terms of u . You will see from this calculation that u does not transform as the time-component of a four vector (that was n); instead, as we will see later, it transforms as the time-time component of a tensor (the stress-energy tensor).

(c) Suppose the sky is uniformly populated with a large number of stars in the unprimed frame, creating a radiation field that has a uniform brightness in all directions. Alternatively, you may think of the cosmic black body radiation, which is very nearly isotropic. The brightness can be defined as the energy per unit area per unit time per unit solid angle. Let B_0 be the constant, uniform brightness in the unprimed frame. Find $B'(\theta')$ in the primed frame, where θ' is the angle between the direction of observation and the x -axis. Find the total brightness, integrated over all solid angles, in the primed frame, compared to $4\pi B_0$, the integrated brightness in the unprimed frame.

3. Jackson problem 11.18.

4. Given a constant \mathbf{E}, \mathbf{B} in one Lorentz frame. Show that there exists a Lorentz frame in which \mathbf{E} and \mathbf{B} are parallel, apart from one exceptional case. Identify the exceptional case.

5. Jackson, problem 11.22.