Reading Assignment: Read Jackson, Chapter 11 to keep up with lectures. Also read pages 8–10 (through Eq. (32.49)) of the notes on the Electromagnetic Field Hamiltonian, which contain a discussion of transverse and longitudinal vector fields.

1. Jackson, problem 5.14 (from last week).


3. A particle with charge $q$ is located at the origin of the coordinates. In the interval $0$ to $T$ the particle is displaced from the origin to $x(T)$ along a path $x(t)$ ($0 \leq t \leq T$). Let $r$ be a point distant from the origin, $r \gg |x(t)|, cT$. The purpose of this exercise is to prove, starting with Maxwell’s equations, that the instantaneous variations of the longitudinal electric field created by charge $q$ at $x$ are exactly compensated by the instantaneous component of the transverse electric field produced by the displacement of the particle.

(a) Calculate, as a function of $x(t)$, the longitudinal electric field $E_\parallel(r, t)$ at point $r$ and time $t$ from charge $q$. Show that $E_\parallel(r, t)$ can be written,

$$E_\parallel(r, t) = E_\parallel(r, 0) + \delta E_\parallel(r, t),$$

where $\delta E_\parallel$ is given by a power series in $|x(t)|/r$. Show that the lowest order term of this expansion can be expressed as a function of $q \dot{x}(t)$ and of the transverse $\delta$-function, $\Delta_\parallel(r)$.

(b) Find the current $J(r, t)$ associated with the motion of the particle. Express the transverse current $J_\perp(r, t)$ at the point of observation $r$ as a function of $q \dot{x}(t)$ and the transverse $\delta$-function $\Delta_\perp(r - x(t))$. Show that to the lowest order in $|x(t)|/r$, one can replace $\Delta_\perp(r - x(t))$ by $\Delta_\parallel(r)$. Write the Maxwell equation giving $\partial E_\perp(r, t)/\partial t$ as a function of $J_\perp(r, t)$ and $B(r, t)$. Begin by ignoring the contribution of $B$ to $\partial E_\perp/\partial t$. Integrate the equation between 0 and $t$. Show that the transverse electric field $E_\perp(r, t)$ produced by $J_\perp(r, t)$ compensates exactly (to lowest order in $|x(t)|/r$) the field $\delta E_\parallel(r, t)$ found in part (a).
(c) By eliminating the transverse electric field between the Maxwell equations for the transverse fields, find the equation of motion for the magnetic field \( \mathbf{B} \). Show that the source term in this equation can be written in a form which only involves the total current \( \mathbf{J} \). Justify the approximation made above of neglecting the contribution of \( \mathbf{B} \) to \( \partial \mathbf{E}_\perp / \partial t \) over short periods \( (T \ll r/c) \).

5. If two events occur at the same time in some inertial frame \( K \), prove that there is no limit on the time separations assigned to these events in other frames, but that their space separation varies from infinity to a minimum which is measured in \( K \).