Physics 209

Fall 2002

Homework 5

Due Friday, October 4 at 5:00pm.

Reading Assignment: Read Jackson, Chapter 11 to keep up with lectures. Also read pages 8–10 (through Eq. (32.49)) of the notes on the Electromagnetic Field Hamiltonian, which contain a discussion of transverse and longitudinal vector fields.

1. Jackson, problem 5.14 (from last week).

2. Jackson, problem 6.1.

3. A particle with charge q is located at the origin of the coordinates. In the interval 0 to T the particle is displaced from the origin to $\mathbf{x}(T)$ along a path $\mathbf{x}(t)$ $(0 \le t \le T)$. Let $\mathbf{r}$ be a point distant from the origin, $r \gg |\mathbf{x}(t)|, cT$. The purpose of this exercise is to prove, starting with Maxwell's equations, that the instantaneous variations of the longitudinal electric field created by charge q at $\mathbf{x}$ are exactly compensated by the instantaneous component of the transverse electric field produced by the displacement of the particle.

(a) Calculate, as a function of $\mathbf{x}(t)$, the longitudinal electric field $\mathbf{E}_\parallel(\mathbf{r}, t)$ at point $\mathbf{r}$ and time t from charge q. Show that $\mathbf{E}_\parallel(\mathbf{r}, t)$ can be written,

$$\mathbf{E}_\parallel(\mathbf{r}, t) = \mathbf{E}_\parallel(\mathbf{r}, 0) + \delta\mathbf{E}_\parallel(\mathbf{r}, t), \tag{1}$$

where $\delta\mathbf{E}_\parallel$ is given by a power series in $|\mathbf{x}(t)|/r$. Show that the lowest order term of this expansion can be expressed as a function of $q\mathbf{x}(t)$ and of the transverse δ-function, $\Delta^\perp_{ij}(\mathbf{r})$.

(b) Find the current $\mathbf{J}(\mathbf{r}, t)$ associated with the motion of the particle. Express the transverse current $\mathbf{J}_\perp(\mathbf{r}, t)$ at the point of observation $\mathbf{r}$ as a function of $q\dot{\mathbf{x}}(t)$ and the transverse δ-function $\Delta^\perp_{ij}(\mathbf{r} - \mathbf{x}(t))$. Show that to the lowest order in $|\mathbf{x}(t)|/r$, one can replace $\Delta^\perp_{ij}(\mathbf{r} - \mathbf{x}(t))$ by $\Delta^\perp_{ij}(\mathbf{r})$. Write the Maxwell equation giving $\partial\mathbf{E}_\perp(\mathbf{r}, t)/\partial t$ as a function of $\mathbf{J}_\perp(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, t)$. Begin by ignoring the contribution of $\mathbf{B}$ to $\partial\mathbf{E}_\perp/\partial t$. Integrate the equation between 0 and t. Show that the transverse electric field $\mathbf{E}_\perp(\mathbf{r}, t)$ produced by $\mathbf{J}_\perp(\mathbf{r}, t)$ compensates exactly (to lowest order in $|\mathbf{x}(t)|/r$) the field $\delta\mathbf{E}_\parallel(\mathbf{r}, t)$ found in part (a).

(c) By eliminating the transverse electric field between the Maxwell equations for the transverse fields, find the equation of motion for the magnetic field **B**. Show that the source term in this equation can be written in a form which only involves the total current **J**. Justify the approximation made above of neglecting the contribution of **B** to $\partial \mathbf{E}_\perp / \partial t$ over short periods ($T \ll r/c$).

5. If two events occur at the same time in some inertial frame K, prove that there is no limit on the time separations assigned to these events in other frames, but that their space separation varies from infinity to a minumum which is measured in K.