Physics 209
Fall 2002
Homework 1
Due Saturday, September 7, 2002

Reading Assignment: By Saturday, August 31, you should have read Chapters 1 and 2 of Jackson. By Saturday, September 7, you should have read Chapters 3 and 4.

1. Jackson, 1.10 (mean value theorem).

2. Jackson, 2.1, part (f) only. In this problem, you are to quote only the numerical answer (in electron volts). Do not present any of your calculations. However, I suggest you do the calculation in whichever system of units you are less familiar with (probably Gaussian), then convert the answer if necessary. Full credit for an answer within 10% of the correct value; otherwise, no credit.

3. Jackson, problem 2.7.

4. Jackson, problem 2.10.

5. Let there be a set of conductors with surfaces, $S_i, i = 1, \ldots, n$. The conductors are not in contact. Let voltages V_i be applied to the conductors. The voltages are measured with respect to ground at infinity.

(a) Explain why the charges Q_i are linear functions of the voltages V_i, that is, show that there exists a matrix C_{ij} such that

$$Q_i = \sum_j C_{ij} V_j. \tag{1}$$

Using Green's reciprocation theorem (see Jackson problem 1.12) show that $C_{ij} = C_{ji}$.

(b) A *positive definite* matrix M_{ij} is a symmetric matrix such that if $\mathbf{a} = (a_1, \ldots, a_n)$ is a vector, then

$$\sum_{ij} M_{ij}\, a_i a_j \geq 0, \tag{2}$$

for all vectors $\mathbf{a}$, with equality if and only if $\mathbf{a} = 0$. Show that C_{ij} is positive definite.

(c) The usual case is $n = 2$. Express the usual capacitance C of two conductors in terms of the components of the 2×2 matrix C_{ij}, and show that it is positive.

6. (Postponed to next week.) Jackson, problem 2.28.