Physics 209  
Fall 2002  
Homework 1  
Due Saturday, September 7, 2002

**Reading Assignment**: By Saturday, August 31, you should have read Chapters 1 and 2 of Jackson. By Saturday, September 7, you should have read Chapters 3 and 4.

1. Jackson, 1.10 (mean value theorem).

2. Jackson, 2.1, part (f) only. In this problem, you are to quote only the numerical answer (in electron volts). Do not present any of your calculations. However, I suggest you do the calculation in whichever system of units you are less familiar with (probably Gaussian), then convert the answer if necessary. Full credit for an answer within 10% of the correct value; otherwise, no credit.


5. Let there be a set of conductors with surfaces, \( S_i, i = 1, \ldots, n \). The conductors are not in contact. Let voltages \( V_i \) be applied to the conductors. The voltages are measured with respect to ground at infinity.

(a) Explain why the charges \( Q_i \) are linear functions of the voltages \( V_i \), that is, show that there exists a matrix \( C_{ij} \) such that

\[
Q_i = \sum_j C_{ij} V_j. \tag{1}
\]

Using Green’s reciprocation theorem (see Jackson problem 1.12) show that \( C_{ij} = C_{ji} \).

(b) A *positive definite* matrix \( M_{ij} \) is a symmetric matrix such that if \( \mathbf{a} = (a_1, \ldots, a_n) \) is a vector, then

\[
\sum_{ij} M_{ij} a_i a_j \geq 0, \tag{2}
\]

for all vectors \( \mathbf{a} \), with equality if and only if \( \mathbf{a} = 0 \). Show that \( C_{ij} \) is positive definite.
(c) The usual case is $n = 2$. Express the usual capacitance $C$ of two conductors in terms of the components of the $2 \times 2$ matrix $C_{ij}$, and show that it is positive.

6. (Postponed to next week.) Jackson, problem 2.28.