1)

$\bar{F} = q(\bar{E} + \bar{v} \times \bar{B})$ in frame $\mathcal{F}$

$\bar{F}' = q(\bar{E}' + \bar{v}' \times \bar{B}')$ in frame $\mathcal{F}'$ (moving at v_0 respect to F)

$\Leftarrow$

$\bar{v}' = \bar{v} - \bar{v}_0$

as both are inertial frames $\vec{F} = \vec{F}'$ (Galileo)

so:

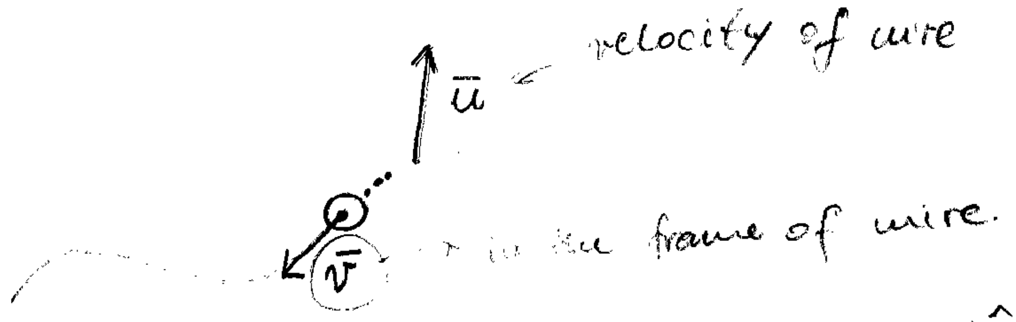
$\bar{F} = q(\bar{E}' + (\bar{v} - \bar{v}_0) \times \bar{B}')$

$= q(\underbrace{E' - (\bar{v}_0 \times \bar{B}')}_{\bar{E}} + \bar{v} \times \underbrace{\bar{B}'}_{})$

$= q(\bar{E} + \bar{v} \times \bar{B})$

$$\boxed{\begin{aligned} \bar{E}' &= \bar{E} + \bar{v}_0 \times \bar{B} \\ \bar{B}' &= \bar{B} \end{aligned}}$$

2)



$\bar{v} \,//\, \hat{\ell}$

$\bar{v} + \bar{u}$ = velocity of electron

magnetic force over electrons

$$F^{e^-}_{mag} = \sum q_i \left[(\bar{v}_i + \bar{u}) \times \bar{B} \right]$$

magnetic force over ions:

$$F^{ions}_{mag} = \sum Q_i (\bar{u} \times \bar{B})$$

Total magnetic work

$$W^{mag} = \underbrace{\sum q_i \left[(\bar{v}_i + \bar{u}) \times \bar{B} \right] \cdot (\bar{v}_i + \bar{u})}_{\emptyset} + \underbrace{\sum Q_i (\bar{u} \times \bar{B}) \cdot \bar{u}}_{\emptyset}$$

$=$

$\boxed{W^{mag} = \emptyset}$ → and this is a general rule.

2) b) In equilibrium

$$\begin{cases} \bar{F}^{e^-}_{w_i} + \bar{F}^{e^-}_{E_i} + \bar{F}^{e^-}_{B_i} + \bar{F}^{e^-}_{R_i} = \bar{\emptyset} \\ \bar{F}^{ion}_{w_i} + \bar{F}^{ion}_{E_i} + \bar{F}^{ion}_{B_i} + \bar{F}^{ion}_{R_i} = \bar{\emptyset} \end{cases}$$

and $\bar{F}_{w_i}$ is a constrain force that forces the electrons to move along the wire; so:

$$\bar{F}^{e^-}_{w_i} = -\left[\underbrace{\bar{F}^{e^-}_{E_i} + \bar{F}^{e^-}_{B_i} + \bar{F}^{e^-}_{R_i}}_{-\bar{F}^{e^-}_{w_i}} - \underbrace{\left(\bar{F}^{e^-}_{E_i} + \bar{F}^{e^-}_{B_i} + \bar{F}^{e^-}_{R_i}\right)}_{\bar{F}^{e^-}_{w_i}} \cdot \hat{\ell}\,\hat{\ell}\right]$$

$$\Downarrow$$

$$\cancel{\bar{F}^{e^-}_{w_i}} = + \cancel{\bar{F}^{e^-}_{w_i}} - \bar{F}^{e^-}_{w_i} \cdot \hat{\ell}\,\hat{\ell} \Rightarrow \boxed{\bar{F}^{e^-}_{w_i} \cdot \hat{\ell} = \emptyset}$$

but the velocity of electrons (in the frame of wire)

$\bar{v}_i \parallel \hat{\ell}$ so $\bar{F}^{e^-}_{w_i} \cdot \bar{v}_i = \emptyset$

so $$\bar{F}^{e^-}_{w_i} \cdot (\bar{u} + \bar{v}_i) = \bar{F}^{e^-}_{w_i} \cdot \bar{u}$$
$$= -\bar{F}^{e^-}_{E_i} \cdot \bar{u} - \bar{F}^{e^-}_{B_i} \cdot \bar{u} - \bar{F}^{e^-}_{R_i} \cdot \bar{u}$$
$$= -q_i \bar{E} \cdot \bar{u} + q_i\left((\bar{u} + \bar{v}_i) \times \bar{B}\right) \bar{v}_i - \bar{F}^{e}_{R_i} \cdot \bar{u}$$

remember that $\bar{F}^{e^-}_{B_i} \cdot (\bar{u} + \bar{v}_i) = \emptyset$

$\bar{F}^{e^-}_{B_i} \cdot \bar{u} = -\bar{F}^{e^-}_{B_i} \cdot \bar{v}_i$

for ions

$$\bar{F}_{ui}^{ion} \cdot \bar{u} = -Q_i \bar{E} \cdot \bar{u} - Q_i (\bar{u} \times \bar{B}) \cdot \bar{u} - \bar{F}_{Ri}^{ion} \cdot \bar{u}$$

the work done by constrains is:

$$\frac{dW^c}{dt} = \sum_i -q_i \bar{E} \cdot \bar{u} + q_i \left((\bar{u} + \bar{v}_i) \times \bar{B} \right) \cdot \bar{v}_i - \bar{F}_{Ri}^{e^-} \cdot \bar{u} +$$

$$\sum_i -Q_i \bar{E} \cdot \bar{u} - \cancel{\$} \quad 0 \quad - \bar{F}_{Ri}^{ion} \cdot \bar{u}$$

$= 0$ because of neutrality

sum 0 ~~xxxx~~ because $\sum_i \bar{F}_{Ri}^{ion} + F_{Ri}^{e^-} = 0$ (is an internal force)

$$W^{constrain} = \sum_i +q_i (\bar{u} \times \bar{B}) \cdot \bar{v}_i + (+q_i) \underbrace{(\bar{v}_i \times \bar{B}) \cdot \bar{v}_i}_{=0}$$

$$\boxed{\left(\frac{dW}{dt}\right)^{constrain} = \sum_i +q_i (\bar{u} \times \bar{B}) \cdot \bar{v}_i}$$ → in a portion of wire

$$\boxed{\left(\frac{dW}{dt}\right)^{c} = +I \oint (\bar{u} \times \bar{B}) \, d\bar{l}}$$ ← along the circuit

$$\left(\frac{dW}{dt}\right)^{total} = \left(\frac{dW}{dt}\right)^{E} + \cancel{\left(\frac{dW}{dt}\right)^{B}}_{\to 0} + \left(\frac{dW}{dt}\right)^{c} + \cancel{\left(\frac{dW}{dt}\right)^{R}}_{\to 0}$$

$\to 0$ because is an internal force.

$$\boxed{\left(\frac{dW}{dt}\right)^{total} = I \oint (\bar{E} + \bar{u} \times \bar{B}) \, d\bar{l}}$$

4) a) In some gauge

$$\bar{B} = \bar{\nabla} \times A \quad \text{with} \quad \nabla^2 \bar{A} = -\bar{J} 4\pi$$

i.e.

$$\begin{cases} * \ \nabla^2 A_x = -J_x \, 4\pi & (1) \\ \nabla^2 A_y = -J_y \, 4\pi & (2) \\ \nabla^2 A_z = -J_z \, 4\pi & (3) \end{cases}$$

$$B_x\big|_{z=0} = \emptyset \Rightarrow \partial_y A_z\big|_{z=0} - \partial_z A_y\big|_{z=\emptyset} = \emptyset \qquad (A)$$

$$B_y\big|_{z=\emptyset} = \emptyset \Rightarrow \partial_z A_x\big|_{z=\emptyset} - \partial_x A_z\big|_{z=\emptyset} = \emptyset \qquad (B)$$

since $B_z\big|_{z=\emptyset} \neq \emptyset$ is given by the functional depende of $A_x(x;y;z=\emptyset)$ and $A_y(x;y;z=\emptyset)$ only

we can add the condition $A_z\big|_{z=0} \equiv \emptyset$

which is a Dirichlet condition for (3)

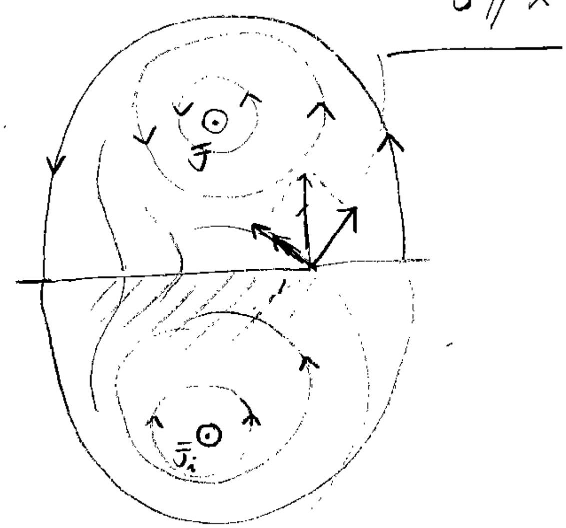
More over $A_z\big|_{z=\emptyset} \equiv \emptyset \Rightarrow \begin{cases} \partial_y A_z\big|_{z=\emptyset} \equiv \emptyset \\ \text{and } \partial_x A_z\big|_{z=\emptyset} \equiv \emptyset \end{cases}$

so (A) and (B) become Neumann conditions of (1) & (2)

$\Rightarrow$ $\bar{A}$ is unique (within the gauge and arbitrary b.c.) $\Longrightarrow$ $\bar{B}$ is absolutelly unique

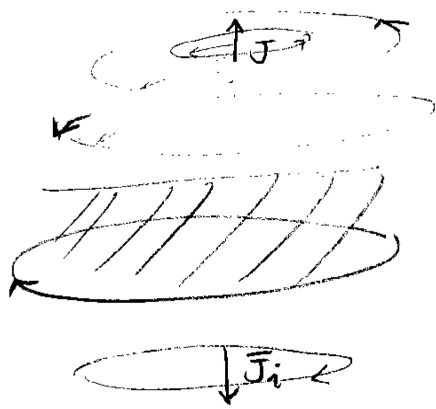
4) ~~a~~ a)

sup. $\bar{J} \parallel \hat{x} \Rightarrow$ ~~$\bar{J}_i = \bar{J}$~~



$$\bar{J}_{i\parallel}(x;y;-z) = \bar{J}_{\parallel}(x;y;z)$$

sup. $\bar{J} \parallel \hat{z}$



← magnetic field created by this differential current has only $\hat{x}$ and $\hat{y}$ components over the plane so

$\bar{J}_i$ must cancel completelly the magnetic field in the plane (i.e. the image current is ~~up~~ in the ~~ot~~ other direction)

$$\bar{J}_{i\perp}(x;y;-z) = \bar{J}_{\perp}(x;y;z)$$

by superposition the general image current $\bar{J}_i$ is

$$\bar{J}_{i\parallel}(x;y;-z) = \bar{J}_{\parallel}(x,y;z)$$

$$\bar{J}_{i\perp}(x;y;-z) = \vec{J}_{\perp}(x;y;z)$$

$$\boxed{\begin{aligned} J_{ix}(x;y;-z) &= J_x(x;y;z) \\ J_{iy}(x,y;-z) &= J_y(x;y;z) \\ J_{iz}(x;y;-z) &= -J_z(x;y;z) \end{aligned}}$$

Notation: $\bar{J}_i(\bar{r}) = \bar{J}(\bar{r} - 2\bar{r}\cdot\hat{n}) - 2\bar{J}(\bar{r} - 2\bar{r}\cdot\hat{n})\cdot\hat{n}$

4) b) $\bar{\nabla}\times\bar{H} = \bar{J}_{free} = \emptyset$

for $z<\emptyset$ the medium is ~~homogeneous~~ homogeneous so

the μ in $\bar{\nabla}\times(\mu\bar{B}) = \emptyset$ can be pulled out

$$\mu\left(\bar{\nabla}\times\bar{B}\right) = \emptyset$$

but $\bar{\nabla}\times\bar{B} = \left(\cancel{\bar{J}_{free}} + \bar{J}_{bound}\right)/\mu$

$$\cancel{\mu}\,\boxed{\frac{\bar{J}_{bound}}{\cancel{\mu}} = \emptyset}$$

[This argument is not valid in the surface because there is a change on μ (in fact a discontinuity)]

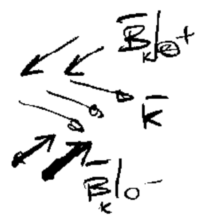
4) c) $\bar{B} = \bar{B}_J + \bar{B}_k$

$\bar{B}_k$: field created by surface current

$\bar{B}_J$: field of current at $z>0$

$$\bar{B}_{J\parallel}\Big|_{z=0} + \bar{B}_{k\parallel}\Big|_{z=0} = 0$$ (desired boundary condition)

if we are close to the surface the field created by the surface it self is:



$$\bar{B}_k\Big|_{z=0^+} = \frac{1}{2}\vec{K}\times\hat{z}\mu_0 = \bar{B}_{k\parallel}\Big|_{z=0}$$

so $$\frac{\mu_0}{2}\bar{K}\times\hat{z} = -\bar{B}_{J\parallel}\Big|_{z=0}$$

remember that $\vec{K} = \vec{K}_\parallel$

$$\overset{A}{\hat{z}}\times(\overset{B}{\bar{K}}\times\overset{C}{\hat{z}}) = -\frac{2}{\mu_0}\hat{z}\times\bar{B}_{J\parallel}\Big|_{z=0} = -\frac{2}{\mu_0}\hat{z}\times\bar{B}_J\Big|_{z=0}$$

$$\bar{K}\underbrace{\cancel{(\hat{z}\cdot\hat{z})}}_{1} - \underbrace{\cancel{(\hat{z}\cdot\bar{K})}}_{0}\hat{z}$$

$$\boxed{\vec{K} = -\frac{2}{\mu_0}\hat{z}\times\bar{B}_J\Big|_{z=0}}$$

Is There a mistake in the problem set? I don't see the error.

~~THE CURRENT~~

4)d) The ~~current~~ field created by surface current is symmetric respect to $z=0$ (an changed in direction z)

so for $z<0$ it is like the ~~images~~ same images ~~where~~ we used to solve $z>0$ were in ~~#~~ $z>0$ and ~~...~~ with the perpendicular component inverted; so they match with ~~the~~ the real currents and duplicate it.

So $$\boxed{\bar{B}(x;y;z) = 2\,\bar{B}_J(x;y;z) \quad \text{for } z<0}$$!

4) e)

$$\bar{F} = \int \left(\bar{K} \times \bar{B}\Big|_{z=0}^{average} \right) ds$$

$$\bar{B}_{z=0}^{average} = \Big[\underbrace{2\bar{B}_J\Big|_{z=0}}_{\text{bellow}} + \underbrace{\bar{B}_J\Big|_{z=0} + \bar{B}_K\Big|_{z=0}}_{\text{above}} \Big] \Big/ 2$$

(see ~~before~~ part c))

$$= \frac{3\bar{B}_J\big|_{z=0} - \bar{B}_J\big|_{z=0}}{2} = \bar{B}_J\Big|_{z=0}$$

so

and $\bar{K} = -\frac{2}{\mu_0} \hat{z} \times \bar{B}_J \Big|_{z=0}$

so

$$\bar{F} = \int -\frac{2}{\mu_0} \left(\hat{z} \times \bar{B}_J\Big|_{z=0} \right) \times \bar{B}_J\Big|_{z=0} \, ds$$

$$\bar{F} = \frac{2}{\mu_0} \int \bar{B}_J\Big|_{z=0} \times \left(\hat{z} \times \bar{B}_J\Big|_{z=0} \right) ds$$

$$\bar{F} = \frac{2}{\mu_0} \int \left(\bar{B}_J\Big|_{z=0} \right)^2 \hat{z} - \left(\bar{B}_J\Big|_{z=0} \cdot \hat{z} \right) \bar{B}_J\Big|_{z=0} \, ds$$

so $$\boxed{F_z = \frac{2}{\mu_0} \int \underbrace{\left(\bar{B}_J\Big|_{z=0} \right)^2 - \left(\bar{B}_J\Big|_{z=0} \cdot \hat{z} \right)^2}_{(\bar{B}_J|_{z=0} \cdot \hat{x})^2 + (\bar{B}_J|_{z=0} \cdot \hat{y})^2 = |\bar{B}_{J//}|_{z=0}|^2} ds}$$

(short way)

5) I will do this ex. starting from the previous calculation (since (20); (22); (28) are not independent results)

from (20) $\quad \bar{D} = \epsilon_0 \bar{E} + \bar{P} - \frac{1}{2}\bar{R}$

by definition $\quad \bar{H} = \frac{\bar{B}}{\mu_0} - \bar{M} - \bar{N} + \dots$

in fact (28) is a def. of $\bar{J}_f + \bar{J}_m$ once the def. of $\bar{D}$ is given.

microscopic Maxwell equation

$$\bar{\nabla} \times \frac{\bar{B}}{\mu_0} = \bar{J} + \epsilon_0 \frac{\partial \bar{E}}{\partial t}$$

inverting (20) and the definition of $\bar{H}$

$$\bar{\nabla} \times \bar{H} + \bar{\nabla} \times \bar{M} + \bar{\nabla} \times \bar{N} = \bar{J} + \frac{\partial \bar{D}}{\partial t} - \frac{\partial \bar{P}}{\partial t} + \frac{1}{2}\frac{\partial \bar{R}}{\partial t}$$

$$\bar{\nabla} \times \bar{H} = \underbrace{\bar{J} - \bar{\nabla} \times \bar{M} - \bar{\nabla} \times \bar{N} - \frac{\partial P}{\partial t} + \frac{1}{2}\frac{\partial \bar{R}}{\partial t}}_{\text{this term is } \bar{J}_m + \bar{J}_f \text{ (both free currents)}} + \frac{\partial \bar{D}}{\partial t}$$

so

$$\boxed{\bar{J} = \langle J \rangle = \bar{J}_m + \bar{J}_f + \bar{\nabla} \times \bar{M} + \frac{\partial P}{\partial t} - \frac{1}{2}\frac{\partial \bar{R}}{\partial t}}$$