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Action-at-a-distance in electrodynamics: the Wheeler–Feynman theory

5.1 Wheeler's modification of Feynman's ideas

Soon after arriving in Princeton for graduate studies from MIT, Feynman learned what was wrong with his idea that the electron does not act upon itself. The matter was that he had missed the radiation damping force. 'When you accelerate an electron it radiates energy and you have to do extra work to account for that energy. The extra force against which this work is done is called the force of radiation resistance. The origin of this force (following the work of the Dutch physicist, Hendrik Antoon Lorentz) was identified in those days as the action of the electron upon itself. The first term of this action, of the electron on itself, gave a kind of inertia (which was relativistically not quite satisfactory). But the inertia-like term was infinite for the point charge. Yet the next term in the sequence gave an energy loss rate which for a point charge agrees exactly with the rate that you get by calculating how much energy is radiated. So, the force of radiation resistance, which is absolutely necessary for the conservation of energy, would disappear if I said that a charge could act on itself.'[1]

Meanwhile Feynman had learned the glaringly obvious fault of his own theory. But he was still in love with the original theory, and was still thinking that with it lay the solution to the difficulties of quantum electrodynamics. 'So, I continued to try on and off to save it somehow. I must have some action develop on a given electron when I accelerate it to account for radiation resistance. But, if I let electrons only act on other electrons the only possible source for this action is another electron in this world. So, one day, when I was working for Professor Wheeler and could no longer solve the problem he had given me, I thought about this again and calculated the following. Suppose I have two charges—I shake the first charge, which I think of as a source, and this makes the second one shake, but the second one shaking produces an effect back on the source. And so I calculated how much that effect back on the first charge was, hoping that it might add up to the force of radiation resistance. It didn't come out right, of course.'[1]

Feynman went to John Wheeler and told him about the ideas he had conceived at MIT and the new difficulty he had encountered in proceeding with his program. Wheeler told him that the answer Feynman got for the problem with the two charges depended upon the charge and the mass of the second charge and would vary inversely as the square of the distance R between the charges, while the force of radiation resistance depended on none of these things. He also pointed out something else that bothered Feynman: 'If we had a situation with many charges all around the original source at roughly uniform density and if we added the effect of all the surrounding charges, the inverse R^2 would be compensated by the R^2 in the volume element and we would get a result proportional to the thickness of the layer, which would go to infinity. That is, one would have an infinite total effect back at the source.'[1] And, finally, he told Feynman that 'when you accelerate the first charge, the second acts later, and the reaction back here at the source would be still later. In other words, the action occurs at the wrong time.'[1] Wheeler told Feynman that what he had described and calculated was just ordinary reflected light, not radiation reaction.

After this critique of Feynman's idea, Wheeler went on to give a lecture in which he worked out the right modification of it. 'First,' he said, 'let us suppose that the return action by the charges in the absorber reaches the source by advanced waves as well as by the ordinary retarded waves of reflected light, so that the law of interaction acts backward in time, as well as forward in time.' But, as Feynman recalled, 'I was enough of a physicist at that time not to say, "Oh, no, how could that be?" For today all physicists know by studying Einstein and Bohr that sometimes an idea which looks completely paradoxical at first, if analyzed to completion in all detail and in experimental situations, may, in fact, not be paradoxical. So, it did not bother me any more than it bothered Professor Wheeler to use advanced waves for the back reaction—a solution of Maxwell's equations which previously had not been physically used.'[1]

Wheeler used advanced waves to get the reaction back at the right time and then he suggested: 'If there were lots of electrons in the absorber, there would be an index of refraction n, so that the retarded waves coming from the source would have their wavelengths slightly modified in going through the absorber. Now, if we shall assume that the advanced waves come back from the absorber without an index . . . , then there will be a gradual shifting in phase between the return and the original signal so that we would only have to figure that the contributions act as if they come from only a finite thickness, that of the first wave [Fresnel] zone. (More specifically, up to that depth where the phase in the medium is shifted appreciably from it would be in vacuum, a thickness proportional to $\lambda/(n-1)$.) Now, the less the number of electrons in here, the less each contributes, but the thicker will be the layer that effectively contributes because with less electrons, the index differs less from 1. The higher the charges of these electrons, the more each contributes, but the thinner the effective layer, because the index would be higher. And when we estimated it (calculated without being careful to keep the correct numerical factor) sure enough, it came out that the action back at the source was completely independent of the properties of the charges that were surrounding the absorber. Further, it was just the right character to represent radiation resistance, but we were unable to see if it was exactly the right size.'[2]

Feynman found this to be 'quite exciting, absolutely terrific!' So, Wheeler said, 'OK, you go home and find out how much advanced and how much retarded waves we need to get the right answer.'[3] Wheeler asked Feynman to figure out what would happen to the advanced effects that you would expect if you put a test charge close to the source. For if all charges generate advanced as well as retarded effects, why would that test charge not be affected by the advanced waves from the source?

'That started it. I found that you get the right answer if you use half-advanced and half-retarded [potentials] as the field generated by each charge. That is, one has to use the solution of Maxwell's equations which is symmetrical in time and the reason why we got no advanced effects at a point close to the source in spite of the fact that the source was producing an advanced field is this: Suppose the source is surrounded by a spherical absorbing wall ten light-seconds away, and that the test charge is one second to the right of the source. Then the source is as much as eleven seconds away from some parts of the wall and only nine seconds away from other parts. The source acting at time $t=0$ induces motions in the wall at time $+10$. Advanced effects from this can act on the test charge as early as eleven seconds earlier, or at $t=-1$. This is just at the time that the direct advanced waves from the source should reach the test charge, and it turns out that the two effects are exactly equal and opposite and cancel out! At the later time $+1$ effects on the test charge from the source and from the walls are again equal, but this time are of the same sign and add to convert the half-retarded wave of the source to full retarded strength.

'Thus, it became clear that there was the possibility that if we assume all actions via half-advanced and half-retarded solutions of Maxwell's equations and assume that the sources are surrounded by material absorbing all the light which is emitted, then we could account for radiation resistance as direct action of the charges of the absorber acting back by advanced waves on the source.'[2]

Feynman devoted several months to check the new theory. He worked hard to make sure that everything was independent of the shape of the container, and that the advanced effects really canceled in every case. Feynman and Wheeler tried to improve the efficiency of their demonstrations, and tried to see more and more clearly how and why their method worked. 'Because of our using advanced waves, we also had many paradoxes, which we gradually reduced one by one, and saw that there was in fact no logical difficulty with the theory. It was perfectly satisfactory.'[2]

In this way, Feynman and Wheeler constructed a classical theory of electrodynamics, which had half-advanced and half-retarded waves in it (classically), and which had no self-energy problem, there being no electron acting on itself. The electrons only acted on each other, and yet they were able to obtain radiation resistance from the fact that there was matter out there which did the absorbing. The next problem was to make a quantum theory of that, and Feynman started to work on it. 'Wheeler kept on giving me little assignments to try to straighten out problems of the classical theory, for he said that the quantum theory was very easy and he knew how to do it, and he kept on putting out announcements that he was going to give a lecture on it, but it always turned out that he didn't quite have it. But I started to work on it anyway, and the problem I had was that the classical theory I was starting out with was not in Hamiltonian form, not the usual form of mechanics, because the action being delayed could be represented beautifully by a minimum principle, with no action, but not by a Hamiltonian because it involved only a Lagrangian (involving only position and velocity). There was no field; it was a direct particle–particle interaction. The only coordinates in the system were [of] the particles, and there was not going to be an infinite number of degrees of freedom.'[3]

It occurred to John Wheeler that Richard Feynman should give a colloquium on their joint work on the classical time-symmetric electrodynamics in the physics department at Princeton. He told Feynman: 'You're a young man, and you should learn how to give talks. You should give the talk and I shall answer the questions. Meanwhile, I'll work on the quantum theory part and give a seminar on that later.' Feynman was frightened. 'It was going to be my first technical seminar, and I was concerned. Wheeler made arrangements with Professor Eugene Wigner, who was the colloquium chairman, to put my talk on the regular seminar schedule. A day or two before the talk I saw Wigner in the hall. "Feynman," he said, "I think your work with Wheeler is very interesting, so I've invited Russell to the seminar." Henry Norris Russell, the great, famous astronomer of the day was coming to the lecture! Wigner went on, "I think Professor von Neumann would be interested." John von Neumann was the greatest mathematician around. "And Professor Pauli is visiting from Switzerland, it so happens, so I've invited Professor Pauli to come"—and by this time I was turning yellow. Finally, Wigner said, "Professor Einstein only rarely comes to our weekly seminars, but your work is so interesting that I've invited him specially, so he's coming too."

'By this time I must have turned green, because Wigner said, "No, no! Don't worry! I'll just warn you though: if Professor Russell falls asleep—and he'll undoubtedly fall asleep—it doesn't mean that the seminar is bad; he falls asleep in all seminars. On the other hand, if Professor Pauli is nodding all the time, and seems to be in agreement as the seminar goes along, pay no attention. Professor Pauli has palsy."'[3,4]

Feynman went to Wheeler and told him about all the big, famous people who were coming to the talk which Wheeler had committed him into giving, and told him how uneasy he felt. Wheeler calmed him by saying that there was no reason to worry, and that he (Wheeler) would answer all the questions. So Feynman prepared his talk, and on the appointed day he did something that young men who have no experience in giving talks often do—'I put too many equations on the blackboard.' For he thought that people in the audience would not be able to follow all the steps on which the theory was based. As he was writing down the equations all over the blackboard ahead of time, Einstein came in and said pleasantly, 'Hello, I'm coming to your seminar, but first, where's tea?' Feynman told him and continued writing on the blackboard. This was not Feynman's first encounter with Einstein; he had gone with John Wheeler to see him about the action-at-a-distance theory that they were developing, and Einstein gave him some hints about the relevant scientific literature on the subject.[3,4]

When everybody came in, Feynman got up and gave the lecture. 'I can still remember very clearly seeing my hands shaking as I was pulling out my notes from a brown envelope, because it was quite a thing to talk in front of all these "monster minds". And then, something happened, which has always happened ever since and is just great: as soon as my mind got on to the physics and trying to explain it, to organize the ideas and how to present them, there was no more worrying about the audience and the personalities! It was all in terms of physics. I was calm, everything was good, I developed the ideas and explained everything to the best of my ability. It was not very good because I was not used to giving lectures, but at any rate there was no more nervousness until I sat down. Then came the questions.'[3,4]

First of all, 'Wolfgang Pauli who was sitting next to Einstein, said: "I do not think this theory can be right because of this, that, and the other thing." It's too bad that I cannot remember what Pauli's reason was for the theory not being right; he may well have hit the nail on the head, but I don't remember what he said, because I was too nervous to listen and I didn't understand the objections. At the end of this criticism, Pauli said to Einstein, "Don't you agree, Professor Einstein? I don't believe this is right, don't you agree, Professor Einstein?" Einstein said, "No," in a soft German voice that sounded very pleasant to me, very polite. "I find only that it would be very difficult to make a corresponding theory [i.e. an action-at-a-distance theory] for gravitational interaction." He meant for the general theory of relativity, which was his baby. After all, he said, general relativity is not so well established as electrodynamics. He continued, "since at this time we do not have a great deal of experimental evidence, I am not absolutely sure of the correct gravitational theory, and with this perspective I would not use that as an argument against you." Einstein appreciated that things might be different from what his theory stated; he was very tolerant of ideas. Very nice, very interesting! I remember that. Then there were some other questions. Wheeler answered Pauli's objections and others, but it was so much like fireworks! Wheeler did answer everything, just as he had promised.'[3,5]

'I wish I had remembered what Pauli said, because I discovered years later that the theory was not satisfactory when it came to making the quantum theory [of it]. It's possible that Pauli noticed the difficulty immediately and explained to me in his question, but I was so relieved that I did not have to answer the questions that I didn't listen to them carefully. I do remember walking up the steps to the Palmer Library with Pauli, who asked me, "What is Wheeler going to say about the quantum theory when he gives his talk?" I said, "I don't know. He hasn't told me. He's working it out himself." "Oh," he said. "The man works and doesn't tell his assistant what's he doing on the quantum theory!" He came closer [to me] and said in a low, secretive voice, "Wheeler will never give that seminar!" And it's true. Wheeler didn't give the seminar. He thought it would be easy to work out the quantum part; he thought he had it, almost. But he didn't. And by the time [his turn] came around, he realized he didn't know how to do it, and therefore, didn't have anything to say. I didn't solve it either—a quantum theory of half-advanced half-retarded potentials—and I worked on it for years.'[3,5]

In a talk given on 21 February 1941, at the meeting of the American Physical Society in Cambridge, Massachusetts, Feynman made a presentation with John Wheeler on their joint work on 'Reaction of the absorber as the mechanism of radiation damping'. In the abstract of their talk, which basically reflected partially the content of Feynman's presentation in the seminar in the fall of 1940, Wheeler and Feynman noted: 'Radiation damping arises from retarded interactions between various parts of an electron of finite size, according to Lorentz. At high frequencies, this damping depends on the electron's structure. Nonelectric stresses are required to hold the electron together. Dirac abandoned this picture and postulated a point electron. Guided by considerations of relativistic invariance he proposed essentially the first term in Lorentz's expression as a possible law for radiation damping. We postulate: 1. that an accelerated point charge in otherwise charge-free space does *not* radiate energy; 2. that in general, the fields which act on a given particle arise only from *other* particles; 3. that these fields are represented by one-half the retarded plus one-half the advanced Liénard–Wiechert solutions of Maxwell's equations. In a universe in which all light is eventually absorbed, the absorbing material scatters back to an accelerated charge a field, part of which is found to be independent of the properties of the material. This part is equal to one-half the retarded *minus* one-half the advanced field generated by the charge. It produces radiative damping (Dirac's expression) and combines with the field of the source to give retarded effects alone.'[6]

Feynman discussed many of these and other ideas with fellow graduate students, like Bill Woodward and John Tukey, all of whom lived together in the Graduate College. At the same time, Feynman continued to develop the general theory of the time-symmetrical action-at-a-distance theory of classical electrodynamics. He tried to write up the work in a variety of different ways. Finally, Wheeler and Feynman appealed to a principle of least action; they represented the theory along the lines of least action instead of Maxwell's equations, because in Maxwell's equations there is one field, and one field would have to act back on its own generator, which is action on itself. But there were so many fields, one for each particle. Unlike Feynman, Wheeler was well familiar with the literature. He knew that an idea like that had been suggested by Frenkel.[7] He also found a paper by Fokker,[8] who had a least action principle, which he showed to Feynman shortly after they began to work on the problem. Fokker had noticed that if you try to write the interaction with the least action you get the advanced and retarded waves, and that's just what Wheeler and Feynman got, half and half, so that they could have an action. Feynman found a way to write Fokker's rather complicated-looking action very simply by using the Dirac delta-function.

Gradually, Wheeler and Feynman got to understand this problem in a variety of ways, until they knew it inside out; they understood all the theorems, all the paradoxes, in fact everything related to these questions, including how to prove everything involved in the clearest possible manner. 'Then Wheeler told me to write it up, and that it should not be more than twenty pages, a simple and beautiful thing. So I wrote it up, and it took me twenty-one pages to discuss and explain everything in a draft manuscript entitled "The interaction theory of radiation".'[3]

Wheeler did not like Feynman's draft of the proposed article. By the time Feynman gave it to him, Wheeler had changed his mind about what he wanted. 'He had decided that the work we had done together was such a grand and wonderful thing that it should be turned into an extensive program of five major papers, something which I did not understand; what I had written was going to be only a part of his [Wheeler's] grand design. In other words, he began by wanting a short concise paper, but ended by conceiving something really elaborate and grand.'[3,9] In the first instance, however, Wheeler reworked and expanded Feynman's draft of the paper and returned him a new, expanded, and unified manuscript in the spring of 1942. By that time, John Wheeler had become engaged upon war-related work at the Metallurgical Laboratory at the University of Chicago, and would not be able to devote any more time to this project until almost the end of World War II. The new manuscript was entitled 'Action at a distance in classical theory: Reaction of the absorber as the mechanism of radiation damping'.[10] Most of it was included in the paper submitted by Wheeler and Feynman to the *Festschrift* published in honor of Niels Bohr's sixtieth birthday.[11]

Soon after his short presentation at the American Physical Society's meeting in Cambridge, Massachusetts,[6] Feynman began to think more seriously about the Ph.D. thesis he had to write. His work on the time-symmetrical action-at-a-distance theory of electrodynamics was not acceptable as a doctoral thesis to John Wheeler, since both he and Wheeler had worked upon this problem together. Feynman and Wheeler continued to discuss problems of mutual interest as they had done before. While all this was going on, Wheeler sent the abstract of a talk he intended to give at the Washington Meeting of the American Physical Society (1–3 May 1941), which was to take place at the National Bureau of Standards and the National Academy of Sciences in Washington, D.C., 'about which he never told me [Feynman]', nor did he say that he was going 'to give a talk at the meeting on the quantum theory of action-at-a-distance' or, for that matter, 'what, if at all, he was going to talk about at Washington.'[3] From the beginning, Feynman's feeling had been that 'we had to straighten out the intrinsic difficulties of classical electrodynamics, before trying to solve the difficulties with quantum electrodynamics, although both of them sounded similar as far as the problem of the self-energy was concerned. Since this difficulty was not cured by quantum mechanics, I thought that we must first cure the difficulties of the classical theory, and then see if the same ideas would apply to quantum physics. Moreover, I never thought that it would be possible to go easily and directly from the classical theory to the quantum theory, but it was essential that we should pursue it in this order if only to see what was required to solve the problem in the two cases. From the beginning Wheeler felt that the transition from the classical case to the quantum case was simple and obvious, but I just did not see how to do it, because he never explained his thoughts about this to me. From time to time he did make some remarks about the transition from the classical to the quantum case, but these were never clear to me. It may be that he had certain ideas which he thought would work but didn't, and I don't know exactly why his announced lecture in the colloquium was canceled.'[3,9]

Feynman did not see the *Bulletin of the American Physical Society*, dated 17 April 1941, in which the program of the Washington meeting was announced a couple of weeks before the actual meeting, and which contained the abstract of Wheeler's talk. This abstract must have been sent several weeks before the meeting, 'so maybe a month and a half after my talk at the Cambridge meeting of the Society, Wheeler must have announced his talk on the quantum theory of action-at-a-distance. I also went to attend the meeting. There he told me that he was going to give a talk, but he still did not tell me what it was going to be: I had to go to listen! Then, in the beginning of his lecture, Wheeler talked a little bit about our ideas about the classical and the quantum theory of action-at-a-distance. Then he changed course, and said: "Concerning this problem," and then he began to talk about the question of van der Waals forces between atoms when they are far apart, farther apart than the wavelength of virtual interactions between them changes from the usual r^{-6} law. He had talked to me about this before, but it was standard electrodynamics; it had no relation to what we had been working upon, though in his initial remarks he brought my name in. But his talk had nothing to do with our work. They were perfectly legitimate things, but I was rather upset that he had done this. He had implied that it had something to do with our theory of action-at-a-distance, but it did not. Perhaps it was not nice of me to do this, but I got up after his talk and remarked that "Professor Wheeler's introduction had nothing to do with the second part of his talk." I wanted to protect myself against the implication that our joint work on the classical theory of action-at-a-distance, when carried into quantum theory, had anything to do with what he presented, for I did not see the connection. Wheeler admitted right away that indeed there was no connection between the two things.'[3,9]

As they walked out together from the lecture room, Wheeler remarked to Feynman, 'I should not have given that talk at all. You are right. I thought that I would have the quantum theory of action-at-a-distance by this time, but I don't. I thought I could talk about this van der Waals thing under that title.'[3,9] The abstract of Wheeler's talk at the Washington meeting did indeed mix up the two things, for it read: '*Action at a distance between simple atoms.* The mutual energy of two atoms in S states varies in a complicated way for small internuclear separations but for larger distances approaches van der Waals $1/r^6$ law. In quantum mechanics this result receives a simple interpretation. . ..'[12]

All this misunderstanding occurred for two reasons: (1) Wheeler did not tell Feynman in advance what the exact topic of his talk at the Washington meeting was going to be, and Feynman had not seen the *Bulletin* listing the detailed program of the meeting. (2) Wheeler had left upon Feynman the impression that he was quite close to unveiling the quantum theory of action-at-a-distance, and would take the first opportunity of announcing it as soon as it was ready. He had already canceled his announced talk at the departmental colloquium at Princeton, and Feynman did not know what to make of his vagueness in this matter. As Feynman recalled: 'I was a little bit unhappy that he could not explain the quantum case, but I think the reason was not that he wouldn't have if he had found a way of doing it. But he didn't quite have it at any point—and the few little attempts he had made to explain it to me, I had shot full of holes right away and had seen his troubles. I think the poor man believed that it was going to be easy, to the point that he would perhaps have it the next morning; so he never told me what it was because he didn't have it until the day of his talk at the meeting, and then he was stuck with the situation. I never felt that he had been trying to do something wrong or dirty to me; I just felt that he mistakenly believed that the answer was just around the corner.'[3,9]

At the Cambridge meeting of the American Physical Society in February 1941, Feynman had already given a summary of their joint work on the classical action-at-a-distance theory. That was the first talk outside Princeton that Feynman was going to give, and the thought of having to give it had scared him. He had written the whole speech out and practiced doing it for the ten prescribed minutes in his room with a friend. 'It took me longer than ten minutes at the meeting, and I heard the bell ring, indicating that my time was up, and I became nervous, then I read the whole speech. So it felt dull, impossible for people to understand, and uninteresting. I must have sounded like a crackpot, and people were bored by listening to me read my speech on and on. At Princeton, I had literally been driven through the wringer at the colloquium; at Cambridge, there were just a few questions, and Wheeler answered one of them.'[3,9]

After the Washington meeting 'Wheeler became engrossed in writing up our work in a grand and wonderful fashion. I began to work on the quantum theory, for I had nothing else to do. Wheeler kept on giving me little problems to explain this or that, and I kept on solving them so fast that it must have driven him mad. Although I started to work on the passage of the classical action-at-a-distance electrodynamics to the quantum case, but the quantum theory was not easy to arrive at, and Wheeler's worry about how we could get the answer quickly was unnecessary.'[3,9]

When the Wheeler–Feynman paper was actually written, Wheeler wrote it.[11] 'It's a long thing, and I didn't write it. I worked with him, but it was not in the spirit in which I thought it should be written. I didn't like it; I didn't think it was a good way to present it; it made things too complicated; there was a much more beautiful way to do it.

'Later on, when I started to talk about quantum theory, I published my own paper. That was in 1948. In my paper on "A relativistic cut-off for classical electrodynamics" [13] I reviewed the classical theory of action-at-a-distance. In my paper on the "Space–time approach to non-relativistic quantum mechanics", I reviewed the 1945 paper with Wheeler again; there I said that this paper was an attempt to quantize the 1945 theory, but that the idea presented then will not work in quantum mechanics, and when I did quantum electrodynamics I was convinced of that.'[3,9]

5.2 The historical context of time-symmetric electrodynamics

Wheeler and Feynman recalled: 'It was the 19th of March 1845 when Carl Friedrich Gauss described the conception of an action-at-a-distance propagated with a finite velocity, the natural generalization to electrodynamics of the view of force so fruitfully applied by Newton and his followers. In the century between then and now what obstacle has discouraged the general use of this conception in the study of Nature?'[14]

The most important physical theory of the nineteenth century was classical electrodynamics, which had its roots in the scientific work of many scientists. From the conceptual point of view, the most important work was the earlier development of electrodynamics by Michael Faraday, who formulated the laws of electrodynamics in terms of electric and magnetic fields and, later, by James Clerk Maxwell's theory of electromagnetic interactions, which gave the complete and unified theory of these phenomena.

Faraday's basic idea was that charged particles interact through an intermediate carrier, namely some new kind of object, called the electric field $\boldsymbol{E}$, responsible for electrical interactions, and another, called the magnetic field $\boldsymbol{H}$, which is responsible for magnetic interactions. The sources of these fields are the charged particles and their currents.

Maxwell's main achievement was the unification of these two vector fields into a general electromagnetic field which, according to the modern relativistic point of view, is a four-tensor field in Minkowski space–time:

$$F^{\mu\nu}(x)=\begin{bmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & -H_3 & H_2 \\ E_2 & H_3 & 0 & -H_1 \\ E_3 & -H_2 & H_1 & 0 \end{bmatrix} \quad (\mu, \nu=0, 1, 2, 3). \tag{5.1}$$

Here $E_{1,2,3}(x)$ are the components of the electric field, and $H_{1,2,3}(x)$ are the corresponding components of the magnetic field. The coordinates of x in Minkowski space–time are: $x_0=ict$, where c is the speed of light and t is the time, and $x_{1,\,2,\,3}$, are the usual space coordinates. Then, in the modern relativistic notation, the first group of Maxwell's equations reads:

$$\frac{\partial F_{\nu\mu}(x)}{\partial x_\mu}=4\pi j_\nu(x) \quad (\nu=0, 1, 2, 3), \tag{5.2}$$

where Einstein's summation rule is assumed (i.e. the summation on repeated indices is to be performed), and the second group of Maxwell's equations is:

$$\frac{\partial F_{\nu\mu}(x)}{\partial x_\rho}+\frac{\partial F_{\rho\nu}(x)}{\partial x_\mu}+\frac{\partial F_{\mu\rho}(x)}{\partial x_\nu}=0 \quad (\nu\neq\mu\neq\rho). \tag{5.3}$$

In equation (5.2) $j_\nu(x)$ is the four-vector of the electric current, the zeroth component of which actually represents the electric charge density. Equation (5.3) may be solved quite simply by introducing the four-vector potential $A_\mu(x)$ such that

$$F_{\nu\mu}(x)=\frac{\partial A_\nu(x)}{\partial x^\mu}-\frac{\partial A_\mu(x)}{\partial x^\nu}. \tag{5.4}$$

Then, as a consequence, instead of equation (5.2), one has to consider the new equation for the four-vector potential:

$$\Box A_\nu(x)=-4\pi j_\nu(x), \tag{5.5}$$

where $\Box=\partial^2/\partial x_\mu\,\partial x^\mu$ is the so-called d'Alembert differential operator or

Dalembertian. In addition, the four-vector potential must satisfy the Lorentz condition:

$$\frac{\partial A_\mu(x)}{\partial x_\mu} = 0. \tag{5.6}$$

The above relations have to hold with certain boundary conditions, which describe the physical conditions at the boundary of the space–time domain in which one has to solve Maxwell's equations. These equations, together with the boundary conditions, give us the complete theory of all electromagnetic phenomena in the macroworld if the equations of motion of the charged particles in the electromagnetic field are also included. Suppose there is a particle with mass m and charge e, and its position four-tensor is a_ν. Then the relativistic four-tensor equation of motion of this particle takes the form

$$mc^2\ddot{a}_\nu = eF_{\nu\mu}(a)\dot{a}^\mu, \tag{5.7}$$

which is identical with the Lorentz equation, in which the three-dimensional force reads $\boldsymbol{F} = e[\boldsymbol{E} + (\boldsymbol{v}/c)\times\boldsymbol{H}]$. In equation (5.7) the dots denote differentiation with respect to the 'proper time' of the particle under consideration. Now we have got a self-consistent theory of electromagnetic interactions of the charged particles, which turns out to be extremely successful for explaining all electromagnetic phenomena in the macroworld, and has a large number of applications in engineering and other practical domains.

It should be mentioned that the whole theory given above may be derived from a single principle of least action $\delta A = 0$ (see Section 6.3), with the action functional

$$A = -\sum_i m_i \int (\dot{a}^\mu_{(i)}\dot{a}_{(i)\mu})^{1/2}\, d\lambda_i - \frac{1}{16\pi}\int F^{\mu\nu}F_{\mu\nu}\, d^4x - \sum_i e_i \int j^\mu A_\mu\, d\lambda_i. \tag{5.8}$$

Here the first term describes the action of the classical particles, and there are supposed to be several of them. This kinetic term only describes the free relativistic particles and it is not connected with their interactions. The index i in all the quantities denotes the particle's number. The second term describes the action of the electromagnetic field $F_{\mu\nu}$ only. It describes the free field without the particles, while the last term in the expression (5.8) describes the interaction of the particles with the field.

But when one tries to extend this theory to the domain of the microworld of elementary particles, one encounters several fundamental difficulties, even at the classical level, before any attempt at the quantization of the theory is made. These difficulties have been thoroughly studied since the times of Alfred Liénard, Emil Wiechert, Max Abraham, and Hendrik Antoon Lorentz. The simplest and most fundamental difficulty is the one connected with the self-energy of the charged point particle, like the electron. One can easily obtain the energy of a spherically symmetric charged particle with radius a and charge e as being of the order of e^2/a. However, if we have a point particle, we have to let the radius a go to zero, and then we obtain an infinite result for the self-energy for the charged point particle. For a moving particle this self-energy appears as a coefficient in some force term, which describes the interaction of the particle with its own field, i.e. the self-interaction of the particle. This term is proportional to the acceleration of the particle and, therefore, may be combined with the left-hand side of equation (5.7). This leads to the 'renormalization' of the mass of the particle, which we represent as $m+\delta m_{\text{selfint}}$, where $\delta m_{\text{selfint}}$ is the mass term which is created by the self-interaction. But $\delta m_{\text{selfint}}$ is proportional to e^2/a and, therefore, goes to infinity for the point particle. Hence, in the classical electrodynamics of a point particle, because of the self-interaction, we are forced to make an infinite renormalization of this particle.

The next difficulty is much more sophisticated. It is connected with the radiation resistance of the moving charged point particle. Suppose we have such a particle, and it describes some trajectory $a_\mu(\mu)$ in Minkowski space–time. Because of the motion of the particle, its electromagnetic field will change, and this disturbance will be propagated in the usual three-dimensional space with the speed of light c, as follows from equation (5.5). The source of the field disturbance is the four-dimensional charged current j_μ in equation (5.5). Liénard and Wiechert[15] obtained the potential of the electromagnetic field of such a particle in a three-dimensional form:

$$A_0(x)=\left[\frac{e}{qR}\right]_{\text{ret}},\quad A(x)=\left[\frac{e\mathbf{v}}{qR}\right]_{\text{ret}}, \tag{5.9}$$

where $q=1-\boldsymbol{v}\cdot\boldsymbol{R}/R$, $\boldsymbol{v}$ is the three-dimensional vector of the particle's velocity at some instant of time τ, $\boldsymbol{R}=\boldsymbol{r}-\boldsymbol{a}$ is the vector from the position $\boldsymbol{a}$ of the particle at the same instant of time τ to the position $\boldsymbol{r}$ of the point in the three-dimensional space, where the potential has to be calculated with the help of equations (5.9), and R is the distance between these two points. Here $\boldsymbol{v}.\boldsymbol{R}$ denotes the scalar product of the three-vectors, hence $\boldsymbol{v}.\boldsymbol{R}/R$ is actually the component of the particle velocity $\boldsymbol{v}$ in the direction of the three-space vector $\boldsymbol{R}$. It seems to be obvious that because of the finite velocity of light, c, the electromagnetic disturbance will spread with some delay in space. Hence, in the formulas (5.9) one ought to calculate the velocity and the position of the particle at the past instant $\tau=\tau_-=t-|\boldsymbol{r}-\boldsymbol{a}|/c$. This retardation means that the disturbance at position $\mathbf{r}$ at time t is caused by the charged particle at another point $\mathbf{a}$, but not at a simultaneous time t, rather at an earlier time τ. The difference between t and τ appears because the disturbance propagates across the intervening distance $R = |\mathbf{r} - \mathbf{a}|$ with a finite velocity c. The subscript 'ret' in equation (5.9) denotes the evaluation of all the quantitites within the brackets at the instant of time τ_-. The corresponding four-tensor $F_{\mu\nu}$ of the electromagnetic field, which one obtains for the Liénard–Wiechert retarded potential (5.9) will be denoted as $F^{\text{ret}}_{\mu\nu}$. This field also includes the field of the particle itself. Using the retarded Liénard–Wiechert potentials, one can see that the charged particle, which moves with some acceleration $\dot{v}$, will radiate electromagnetic waves. But if such a particle radiates energy in the form of electromagnetic waves, it must suffer a damping in its mechanical energy. The magnitude f_{damp} of this damping force is found to be

$$f_{\text{damp}} = \tfrac{2}{3}(e^2/c^2)\ddot{v}. \qquad (5.10)$$

The coefficient $\frac{2}{3}e^2/c^3$ is related to the so-called relaxation time of the particle, which is usually very small; for an electron this relaxation time is about 10^{-23} seconds. This time is typical of the radiation process. The origin of this force lies in the self-interaction of the charged particles with their own fields. This self-interaction allows a charged particle to radiate energy in the absence of other charges or fields in the entire space. This radiation is a well-verified experimental phenomenon. The force given by equation (5.10) also gives a correct account of the conservation of energy, momentum, and angular momentum of the particle.

Unlike the usual forces in classical physics, the damping force depends on the second derivative $\ddot{v}$ of the velocity of the particle, i.e. of the third derivative $\dddot{a}$ of its position. This fact leads to very strange physical consequences: equation (5.7) has more solutions than are needed. For example, if there are no external forces, this equation leads to two completely different solutions. The first solution is of the usual type and shows that the free particle will preserve its velocity unchanged for an infinitely long time; for instance, it may stay at rest all the time. But the other solution shows that, because of the self-interaction, the particle, which is in a state of rest at the initial instant of time, will run away with an exponentially increasing acceleration. The solution yielding such self-acceleration is clearly unphysical and has to be excluded with the help of certain additional physical conditions.

P. A. M. Dirac proposed to eliminate such runaway solutions by fixing a special value of the initial acceleration.[16] But this approach leads to a new difficulty, called *preacceleration*. It turned out that under Dirac's condition the particle must accelerate *before* the force is applied. This notion violates the usual conception of causality. However, such acausal behavior takes too small a time to be observable. It is of the order of the time required by light to cross the distance of about the 'classical radius' of an elementary particle; hence, a quantum description becomes necessary.

This was the situation in classical electrodynamics following Lorentz's work on his old theory of the electron. The first attempts at a quantization of this theory led to new difficulties besides the old ones.†

† In this section, we have explained the Liénard–Wiechert retarded potential in equation (5.9), which gives the retarded solutions of equation (5.5) and describes the retarded solution $F^{\text{ret}}_{\mu\nu}$ of Maxwell's equations (5.2) and (5.3). But these equations also have another type of solutions, the advanced ones, which one can obtain by using formulas (5.9) but at another instant of time, $\tau=\tau_+=t+|\mathbf{r}-\boldsymbol{a}|/c$:

$$A_0(x)=\left[\frac{e}{qR}\right]_{\text{adv}}, \qquad A(x)=\left[\frac{e\mathbf{v}}{qR}\right]_{\text{adv}}. \tag{5.11}$$

At first, these solutions look quite strange. Nevertheless, given the finite velocity of light, the electromagnatic disturbance will spread with some advance in space. In the formulas (5.11) one ought to calculate the velocity and the position of the particle at the advanced instant τ_+. This means that the disturbance at position **r** at time t is caused by the charged particle at another point $\boldsymbol{a}$, not at a simultaneous time t, but rather at a *later* time τ_+. The disturbance propagates across the intervening distance $R=|\mathbf{r}-\boldsymbol{a}|$ in *advance*. The subscript 'adv' in equation (5.11) denotes the evaluation of all quantities in the brackets at the later instant τ_+. The corresponding four-tensor $F_{\mu\nu}$ of the electromagnetic field, which one obtains for the advanced potential (5.11) will be denoted by $F^{\text{adv}}_{\mu\nu}$. The advanced solutions appear to violate the usual notions of causality, because the cause of the disturbance lies in the future.

However, one cannot ignore the advanced solutions of Maxwell's equations in an *ad hoc* fashion. Actually, the general solution of these equations is a linear combination of the advanced and the retarded solutions: $F_{\mu\nu}=kF^{\text{adv}}_{\mu\nu}+(1-k)F^{\text{ret}}_{\mu\nu}$, where k is an arbitrary constant. The choice of this constant depends on the so-called boundary conditions.

Suppose we consider a collection of charged particles in some volume V bounded by the surface S. Then there will exist electromagnetic waves of two kinds in the volume V. One kind are waves that are caused by charges within the volume V, and they spread through the surface S outside the volume.

The other kind of electromagnetic waves are coming in from the space around this volume. These waves are caused by outside charges and they spread through the volume V within the surface S. If we wish to obtain only the retarded waves within the volume V, then it is necessary to choose the boundary condition on the solutions of Maxwell's equations, which eliminates the incoming waves. This boundary condition is known as Sommerfeld's radiation condition. It eliminates the advanced solutions of Maxwell's equation in the volume V in accordance with our experience in the real world. Sommerfeld's radiation condition seems to be very natural if the volume V includes the entire universe. Wheeler's modification of Feynman's idea to connect the radiation resistance of the electron with the reaction of the absorbers at a long distance from the radiating particle, in this sense, means a change of boundary conditions for the solutions of Maxwell's equations. Instead of Sommerfeld's radiation condition, Wheeler proposed to put an absorber on the boundary of the universe, and then to explain radiation resistance as a back reaction of the absorber on the radiating particle. It turned out that one can satisfy this new boundary condition with a proper choice of the constant k in a linear combination, which gives the general solution of Maxwell's equations. Feynman established that the new boundary condition will be fulfilled if one put $k=\frac{1}{2}$, that is, if a one-half retarded and one-half advanced mixture is taken as a solution of Maxwell's equations. Feynman considered the simplest model of the universe: namely, flat Minkowski space-time. It turns out that in modern relativistic models of the universe the new Wheeler–Feynman boundary condition gives some new and interesting results in cosmology.[17]

5.3 Electrodynamics without the electromagnetic field: the Wheeler–Feynman theory

In Section 5.1, we mentioned that in the spring of 1941 Richard Feynman wrote the draft of a paper entitled 'The interaction theory of radiation', in which he gave a summary of the work on an action-at-a-distance theory of classical electrodynamics, which he and John Wheeler had carried out during the previous few months. He stated the fundamental assumptions of this theory as follows:[18]

'(1) The acceleration of the point charge is due only to the sum of its interactions with other charged particles (and to "mechanical forces"†). A charge does not act on itself.

'(2) The force of interaction which one charge exerts on the second is calculated by means of the Lorentz force formula,‡ in which the fields are the fields generated by the first charge according to Maxwell's equations.

'(3) The fundamental (microscopic) phenomena in nature are symmetrical with respect to interchanges of past and future.§

'(4) The limit of the velocity of each charge for increasingly remote (past or future) times is less than the velocity of light.

'According to the second assumption alone, the force exerted by one charge on a second might be obtained from the field derived from the retarded potentials of Liénard and Wiechert. Thus the second charge would be affected by an amount determined by the *previous* motion of the first charge. This is not the only possibility, however; one could, for example, use the advanced potentials. In this case, the second charge would be affected by an amount depending on the *later* motion of the first charge. The requirement that the effects be unchanged if one interchanges past and future removes the ambiguity and demands that one utilize one-half the retarded plus one-half the advanced potentials to calculate the force on the second point charge due to the first. This is exactly the law of interaction that one derives from the

† The present theory is one to describe those phenomena which are usually considered to be due to electromagnetic effects. Forces on charged particles such as nuclear forces on protons, or "quantum effects" on electrons, will be classified as "mechanical" forces and will not be discussed further in this paper.

‡ Force $= e[\boldsymbol{E} + (\boldsymbol{v}/c) \times \boldsymbol{H}]$.

§ The original statement was 'The fundamental equations are to be invariant with respect to interchange of sign of the time in them (symmetrical with respect to interchange of past and future),' which Feynman changed.

principle of least action of Fokker, and that principle may well have formed the starting point of this theory.'[18]

Feynman then proceeded to discuss the applications of this principle to certain idealized situations in order to get an idea of its physical meaning. In the first place, he noted that a single accelerating charge in otherwise charge-free space will radiate no energy. There can be no radiation damping, since there are no electrodynamic fields acting on the charge, no other charges being present to generate such fields. Feynman then presented the explanation which he and Wheeler had devised for the mechanism of radiation damping in their theory.

Wheeler reworked Feynman's manuscript and returned to him the new expanded version in the spring of 1942. The title of the new expanded version was: 'Action at a distance in classical theory: reaction of the absorber as the mechanism of radiation damping'.[10] War work interrupted the further development of the Wheeler–Feynman theory until 1945, when a second version of the paper was submitted to the *Festschrift* published in honor of Niels Bohr's sixtieth birthday.[11] In Wheeler's long-range plan to overhaul electrodynamics completely, this paper was intended to be Part III of a series of five papers. Part II, which should have preceded the 1945 paper, was published in 1949.[19] At about the same time, the arrival of the new quantum electrodynamics, formulated by Feynman, Schwinger, and Tomonaga, made Wheeler's program redundant.

In the 1945 paper in the *Reviews of Modern Physics*, Wheeler and Feynman gave a simple and elegant explanation of radiation damping. Suppose, there are N charged particles and each of them is the source of some advanced field $F^{(k)}_{\text{adv}}$ and some retarded field $F^{(k)}_{\text{ret}}$, with k being the number of the particle. So long as there is no self-interaction, the field acting on the ath particle is given, according to the theory of action-at-a-distance, by the sum

$$\sum_{k \neq a} \left(\tfrac{1}{2}F^{(k)}_{\text{ret}} + \tfrac{1}{2}F^{(k)}_{\text{adv}}\right). \tag{5.12}$$

This field can be broken down into three parts:

$$\sum_{k \neq a} F^{(k)}_{\text{ret}} + \left(\tfrac{1}{2}F^{(a)}_{\text{ret}} + \tfrac{1}{2}F^{(a)}_{\text{adv}}\right) - \sum_{\text{all } k} \left(\tfrac{1}{2}F^{(k)}_{\text{ret}} - \tfrac{1}{2}F^{(k)}_{\text{adv}}\right). \tag{5.13}$$

Of these terms the third vanishes for the complete absorber. In the case of nonrelativistic velocities, the third term had been shown by Dirac[16] to give just the Lorentz damping force, given by equation (5.10). The second term gives rise to the phenomenon of radiation damping. Thus, the charged particle interacts effectively only with the retarded field of the other charged particles, which is expressed by the first term in the expression (5.13). In addition, this particle experiences a Lorentz damping force due to its own acceleration.

At first, the last conclusion seemed to be slightly confusing. Wheeler and Feynman had started from a completely time-symmetric theory, which was time-reversible. But they arrived at a result which is obviously time-irreversible, since the interaction turns out to be effectively retarded. The explanation of this difficulty lies in thermodynamical and statistical considerations. The particles in the absorber, in accordance with thermodynamical laws, will have random motion and the absorber would tend to go from ordered to disordered states of higher entropy. For this reason, the prevalence of the retarded over the advanced potentials will be observed. In his initial draft of the article, Feynman had written:

'It may be worthwhile to make a few remarks at this point about the irreversibility of radiative phenomena. We must distinguish between two types of irreversibility. A sequence of natural phenomena will be said to be microscopically irreversible if the sequence of phenomena reversed in temporal order in every detail could not possibly occur in nature. If the original sequence and the one reversed in time have a vastly different order of probability of occurrence in the macroscopic sense, the phenomena are said to be macroscopically irreversible.

'The Lorentz theory predicts the existence of microscopically irreversible phenomena in systems which are not closed (for example, energy is always lost by the system to empty space as radiation). In our theory the phenomena are microscopically reversible in any system. It seems at first sight paradoxical that the two theories can ever lead to the same result, as they do in closed systems. The reason is that the phenomena predicted for closed systems are actually reversible even within the framework of the Lorentz theory which uses only retarded waves.† The apparent irreversibility in a closed system, then, either from our point of view or the point of view of Lorentz, is a purely macroscopic irreversibility. We believe that all physical phenomena are microscopically reversible, and that, therefore, all apparently irreversible phenomena are solely macroscopically irreversible.'[18]

The main obstacle toward the realization of Feynman's MIT program (see Chapter 4) was thus overcome in the Wheeler–Feynman absorption theory of radiation of the charged particles. They had succeeded in completely explaining the force of radiation resistance in the theory without self-interaction. However, one further step was needed for overcoming the difficulty with the infinitely many degrees of freedom in the field, which still remained in the theory of one-half retarded plus one-half advanced waves. In accordance with the expression (5.12), one has to write down the equation of motion of the charged particle, with particle number a, in the form

$$mc^2\ddot{a}_\nu = e \sum_{k \neq a} \left(\tfrac{1}{2}F^{(k)}_{\mathrm{ret}} + \tfrac{1}{2}F^{(k)}_{\mathrm{adv}}\right)\dot{a}^\mu. \qquad (5.14)$$

Now the question was: Is it possible to write down some new equations of

† 'That this and the following statement are true in the Lorentz theory was emphasized by Einstein in a discussion with Ritz.[20] Our viewpoint on the matter is essentially that of Einstein. (We should like to thank Prof. W. Pauli for calling our attention to this discussion.)'

motion of the particles, which do not include the electromagnetic field, but nevertheless have the same solutions as equations (5.14)? This question was quite old, and had its roots in Newton's theory of direct interaction between particles. The most well-known example of a direct interaction between particles at some distance is Newton's law of gravitational interaction, which says that the gravitational force between two particles with masses m_1 and m_2 equals $m_1 m_2/|\boldsymbol{r}_1 - \boldsymbol{r}_2|^2$ up to some coefficient, called Newton's gravitational constant. One can see that the particles interact, being at the distance $|\boldsymbol{r}_1 - \boldsymbol{r}_2|$. This interaction spreads infinitely fast, because if one particle changes its position, given by the corresponding vector $\boldsymbol{r}$, then the force, which acts on the other particles changes immediately together with the distance $|\boldsymbol{r}_1 - \boldsymbol{r}_2|$. No field is required to describe such an interaction. But the infinitely large velocity of the propagation of interaction is in contradiction with experiments and with Einstein's theory of relativity; therefore, one has to take into account the finite velocity of the spreading of the interaction to avoid this difficulty.

Since the work of Maxwell, it was known that the velocity of propagation of the electromagnetic interaction is the velocity of light. Hence one needs some modification of the action-at-a-distance theory to explain the finite velocity of the propagation of electrodynamic interaction without the notion of the field. Wheeler and Feynman took note of this problem by saying:

'It is easy to see why no unified presentation of classical electrodynamics has yet been given, though the elements for such a description are all present in isolated form in the literature. The development of the electromagnetic theory came before the era of relativity. Most minds were not prepared for the requirement that interactions should be propagated with a certain characteristic speed, still less for the possibility of both advanced and retarded interactions. Newtonian instantaneous action-at-a-distance with its century and a half of successes seemed the natural framework about which to construct a description of electromagnetism. Attempt after attempt failed.[21] And unfortunately uncompleted was the work of Gauss, who wrote to [Wilhelm] Weber on 19 March 1845: "I would doubtless have published my researches long since were not at times I gave them up; I had failed to find what I regarded as the keystone, *Nil actum reputans si quid superesset agendu*: namely, the derivation of the additional forces—to be added to the interaction of electrical charges at rest, when they are both in motion—form an action which is propagated not instantaneously but in time as is the case with light."[22] These failures and the final success via the apparently quite different concept of field were taken by physicists generally as convincing arguments against electromagnetic action-at-a-distance.

'Field theory taught gradually for over seven decades difficult lessons about constancy of light velocity, about relativity of space and time, about advanced and retarded forces, and in the end made possible by this circuitous route the theory of interparticle interaction which Gauss had hoped to achieve in one leap. On this route and historically important was Liénard[23] and Wiechert's[24] derivation from the equations of Maxwell of an expression for the elementary field generated by a point charge in arbitrary state of motion. With this expression as the starting point Schwarzschild arrived at a law of force between two point charges which made no reference to field quantities. Developed without benefit of the concept of relativity, and expressed in the inconvenient notation of the prerelativistic period, his equations made no appeal to the physicists of the time. After the advent of relativity Schwarzschild's results were rederived independently by Tetrode and Fokker. These results are most conveniently summarized in Fokker's principle of stationary action.'[25] According to this principle, $\delta A_F = 0$, where Fokker's action functional A_F is

$$A_F = -\sum_i m_i \int (\dot{a}_i^\mu \dot{a}_{(i)\mu})^{1/2}\, d\lambda_i$$

$$-\tfrac{1}{2} \sum_{i \neq j}\sum e_i e_j \iint \delta((a_{(i)} - a_{(j)})^2)\dot{a}^\mu_{(i)} \dot{a}_{(j)\mu}\, d\lambda_i\, d\lambda_j. \quad (5.15)$$

Here δ denotes Dirac's delta-function, and its argument is the square of the relativistic four-dimensional distance between the space–time points of the particles with numbers i and j: $[(a^\mu_{(i)} - a^\mu_{(j)})]^2 = (a_{(i)\mu} - a_{(j)\mu})$. This means that in terms of four-dimensional space–time the particles interact only at zero four-dimensional distance. In more physical terms, this fact means that two particles interact when, and only when, their locations in space–time can be connected by a light ray.

Fokker's action, equation (5.15), looks quite different compared to the usual electrodynamical action given by the expression (5.8). Fokker's action obviously depends only on the particle's variables, not on the field's variables. Thus the electromagnetic field is eliminated from the theory with the action given by equation (5.15). Feynman recalled: 'We also found that we could reformulate [the new theory] in another way, and that is by [means of] the principle of least action. Since my original plan was to describe everything directly in terms of particle motions, it was my desire to represent this new theory without saying anything about fields. It turned out that we found a form for an action directly involving the motions of charges only, which upon variation would give the equations of motion of these charges.'[3,9]

Wheeler and Feynman noted: 'However unfamiliar this direct interparticle treatment compared to the electrodynamics of Maxwell and Lorentz, it deals with the same problem, talks about the same charges, considers the interaction of the same current elements, obtains the same capacities, predicts the same inductances, and yields the same physical conclusions. Consequently action-at-a-distance must have a close connection with field theory. But never does it consider the action of a charge on itself. The theory of direct interparticle action is equivalent, not to the usual field theory, but to a modified or *adjunct field theory. . . .*'[25]

Wheeler and Feynman actually showed that if one were to write down the second term in the action equation (5.15) in the form

$$\frac{1}{2}\sum_{i\neq j}\sum e_i\int \dot{a}^{\mu}_{(i)}A_{(j)\mu}\,d\lambda_i, \qquad (5.16)$$

where

$$A_{(j)\mu}=e_j\int \delta((a_{(i)}-a_{(j)})^2)\,da_{(j)\mu} \qquad (5.17)$$

is the electromagnetic potential created by the charged particle with number j, then Maxwell's equations will be satisfied together with all other conditions and equations in the Feynman–Wheeler modification of electrodynamics. 'So, all of classical electrodynamics was contained in this very simple form. It looked good and, therefore, it was undoubtedly true, at least to the beginner. It automatically gave half-advanced and half-retarded effects and it was without fields. By omitting the term in the sum when $i=j$, I omit self-interaction and no longer have any infinite self-energy. This then was the hoped-for solution to the problem of ridding classical electrodynamics of infinities. . ..

'I would like to make the following remark. The fact that electrodynamics can be written in so many ways—the differential equations of Maxwell, various minimum principles with fields, minimum principles without fields, all different kinds of ways—was something I knew but have never understood. It always seems odd to me that the fundamental laws of physics, when discovered, can appear in so many different forms that are not apparently identical at first, but with a little mathematical fiddling you can show the relationship. An example of that is the Schrödinger equation and the Heisenberg formulation of quantum mechanics. I don't know why this is—it remains a mystery, but it was something I learned from experience. There is always another way to say the same thing that doesn't look at all like the way you said it before. I don't know what the reason for this is. I think it is somehow a representation of the simplicity of nature. A thing like the inverse square law is just right to be represented by the solution of Poisson's equation, which, therefore, is a very different way to say the same thing that doesn't look at all like the way you said it before. I don't know what it means, that nature chooses these curious forms, but maybe that is a way of defining simplicity. Perhaps a thing is simple if you can describe it fully in several different ways without immediately knowing that you are describing the same thing.

'I was now convinced that since we had solved the problem of classical electrodynamics (and completely in accordance with my program from MIT, with only the direct interaction between particles, in a way that made fields unnecessary) everything was definitely going to be all right. I was convinced that all I had to do was make a quantum theory analogous to the classical one and everything would be solved.'[26]

Thus Feynman's MIT program of modifying classical electrodynamics was successfully fulfilled. Besides this fundamental development, Feynman arrived at some new physical points of view, different from the customary ones. The first was the new approach to physical systems. 'In the customary view, things are discussed as a function of time in very great detail. For example, you have the field at this moment, differential equation gives you the field at the next moment, and so on—a method which I shall call the Hamiltonian method, the time differential method. We have, instead (in [(5.15)], say) a thing that describes the character of the path throughout all space and time. The behavior of nature is determined by saying her whole space–time path has a certain character. For an action like [(5.15)] the equations obtained by variation of [A_F] are no longer at all easy to get back into Hamiltonian form. If you wish to use as variables only the coordinates of the particles, then you can talk about the property of the paths—but the path of one particle at a given time is affected by the path of another at a different time. If you try to describe, therefore, things differentially, telling what the present conditions of the particles are, and how these present conditions will affect the future—you see, it is impossible with particles alone, because something the particle did in the past is going to affect the future.

'Therefore, you need a lot of bookkeeping variables to keep track of what the particle did in the past. These are called field variables. You will also have to tell what the field is at this present moment, if you are to be able to see later what is going to happen. From the overall space–time view of the least action principle, the field disappears as nothing but bookkeeping variables insisted on by the Hamiltonian method.'[27] This new point of view will play an essential role in Feynman's invention of the path integral method for the quantization of classical systems (see Chapters 6 and 10).

As a by-product of this point of view at about the same time, in the fall of 1940, Feynman received a telephone call from John Wheeler at the Graduate College in Princeton, in which he said that he knew why all electrons have the same charge and the same mass. 'Why?' asked Feynman, and Wheeler replied, 'Because they are all one and the same electron.'[3,26]

One usually describes the motion of the particles as world-lines in space-time. These world-lines depend on the change of the coordinates and proper times of the particles. One usually supposes that a particle's world-line goes only from the past to the future. Then, every such world-line describes one particle at a given instant of time. If there are several such world-lines at a given instant of time, this would mean that there are several such particles, just as many as the number of world-lines at that instant (see Fig. 5.1).

Wheeler said to Feynman: 'Suppose that the world-lines which we were ordinarily considering before in time and space, if only going up in time, were a

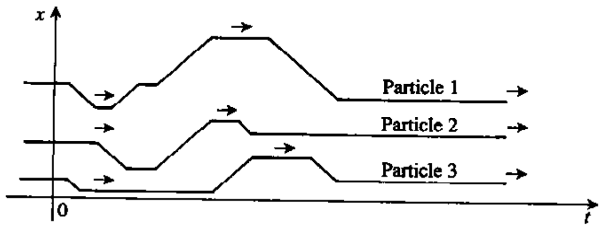

Fig. 5.1. The world-lines of three particles, which move ahead in time.

tremendous knot, and then, when we cut through the knot by a plane corresponding to a fixed time, we would see many, many world-lines and that would represent many electrons [see Fig. 5.2]—except for one thing. If in one section of this is an ordinary electron world-line, in the section in which it reversed itself and is coming back from the future we have the wrong sign to the proper time—to the proper four-velocities—and that's equivalent to change of sign of the charge,† and therefore that part of the path would act like a positron.'[26]

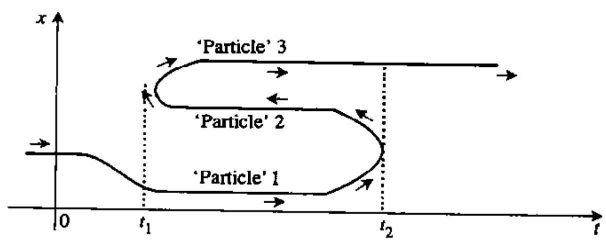

Fig. 5.2. The world-line of a particle, which turns back in the time instant t_2 and then at time instant t_1 turns in the forward direction in time. The observer will see three particles between the instances t_1 and t_2.

Feynman did not take 'the idea that all electrons were the same one from (Wheeler) as seriously as the observation that positrons could simply be represented as electrons going back from the future to the past in a back section of their world lines.'[26] Later on, in his theory of positrons, Feynman

† Indeed, if the increasing of the proper time has the wrong sign, then, as a consequence, we shall have a wrong sign of the four-velocity $v = \dot{a}$ and of the four-acceleration $\ddot{a}$ of the particle. But according to equation (5.7) we can include this wrong sign in the charge e of the particle, to which the right-hand side of this equation is proportional. Hence, in this part of the world-line, which goes back in time, one can think that there is one particle with the right sign for the increase of the particle's proper time, but the opposite sign of the electric charge. The mass of this imaginary particle and all its other characteristics will be the same, and we can conclude that the electron, going back in time, will behave just like a positron.

would make use of this idea in order to avoid Dirac's hole theory of positrons (see Chapter 13). However, in the conversation that fall in 1940, he said to Wheeler, 'But, Professor, there aren't as many positrons as electrons,' and Wheeler replied, 'Well, maybe they are hidden in the protons or something.'[26] Feynman's objection was based on the obvious fact (see Fig. 5.2) that each time an electron reverses its world-line in time, it will go back for some time, and then it must return to the forward direction of time. According to Wheeler's idea, the positron is part of the electron's world-line, which goes back in time. Hence, every 'positron' will correspond to another part of the world-line in the right direction, which represents another electron. Hence, if we have only one electron in the whole universe, and those positrons are parts of its world-line, which travel in the reversed time direction, we must have at each instant as many positrons as electrons minus one. So, if one were to take Wheeler's idea seriously, one needs some speculation to explain where all the positrons are.

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